## PARALLEL PULSED JETS FOR PRECISE UNDERWATER PROPULSION

by

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#### Abstract

A significant limitation for underwater robots is their ability to maneuver in tight spaces or for complex tracking tasks. Next generation vehicles require thrusters that can overcome this problem and efficiently provide high maneuverability at low speeds. Recently, thruster design has begun to draw inspiration from nature's swimmers, applying the principles of pulsed jet propulsion to robotic thrusters. Although most developments have focused on single jet actuators, nature provides some indications that multi-jet systems can provide propulsive benefits – marine invertebrates called salps connect into chains of individual animals that each eject short jets to collaboratively move the entire chain efficiently around the ocean. However, despite the promise of multi-jet propulsion, there are no existing models or empirical data that explain the physics of multi-jet propulsion. As a result, there are no physicallymotivated rules to guide the design of man-made multi-jet thrusters.

In this thesis, I experimentally investigate how interactions between neighboring jets in a multi-jet thruster will affect the system's propulsive performance. I use high-speed fluorescence imaging to investigate the mutual influence of two pulsed jets under conditions relevant to low-speed maneuvering in a vehicle ( $Re \sim 350$ ). Using a new force estimation technique developed in this thesis, I analyze the video data to evaluate how thrust and efficiency are affected by the jet spacing. This analysis reveals that, compared to non-interacting jets, the efficiency and thrust generated by the pair of interacting jets can fall by nearly 10% as the jets are brought into close proximity. Based on this data, I develop a model of vortex interactions to explain the thrust and efficiency drop. The data and model described in this thesis contribute new insights to understand vortex formation in pulsed jets, and these results can be used to guide the design of multi-jet underwater propulsion systems.

Thesis Supervisor: Douglas P. Hart Title: Professor of Mechanical Engineering

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# Contents

1	Introduction	15
2	Background         2.1       Thrust generation with pulsed jets         2.1.1       General Principles         2.1.2       Factors that affect pulsed jet development         2.2       Vortex ring interactions	<b>19</b> 19 19 21 24
3	Experimental Framework3.1Experimental apparatus3.2Derivation of thrust estimation technique3.3Measurements & processing	<b>27</b> 27 32 37
4	Experiment Results4.1Analysis of single-jet thrust production4.2The effect of two-jet interactions on thrust4.3Effects on efficiency	<b>45</b> 46 50 60
5	Discussion of two-nozzle interactions in pulsed jet formation 5.1 Sources of nozzle over-pressure during vortex formation 5.2 Geometric argument for thrust reduction based on the slug model 5.3 Scaling argument for the functional form of thrust reduction: $T \sim T_{\infty}(1 - C\rho\tilde{\Delta}^{-6})$	<b>61</b> 62 66
	5.4 Final remarks on the scaling of $T$ with $\tilde{\Delta}$	71 72 73
6	Summary and Conclusions	77
Aŗ	Appendices	
A	Pressure distribution around a translating and growing ellipsoid	81
в	A pressure-based model to describe two-jet interaction	85
$\mathbf{C}$	Additional data from two-nozzle experiments	89

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# List of Figures

3-1 Experimental setup. (a) Experiment schematic depicting all of the system components. The side view reveals the illumination and imaging system, which images the jets from the front. (b) Image of backlit nozzles revealing the field of view and geometry. (c) Typical fluorescence image of the pulsed jet wake during an experiment.

28

29

- 3-2 Volume flux through the nozzle as measured by the flow meters attached to each reservoir. The volume flow through the nozzles is approximately linear in time, so to calculate the volume flow rate, the data for each nozzle are fit to a line. Here, the volume flow rate through each nozzle is  $\dot{Q} = 1.68 \pm 0.04 \text{mL/s.}$
- 3-3 A sequence of images at different points in the jet's early development. These frames correspond to a single jet. Not only does the center of massmove as more fluid is ejected, but the volume grows and the wake's shape evolves with time. These frames are only a subset of a typical experiment; for the full analysis, video data is acquired every 2.5ms.
- Estimating thrust based on wake evolution. (a) The control volumes 3-4around a model vehicle and its vortical wake. The thrust experienced by the vehicle is balanced by the rate of momentum change in the wake. (b) A detailed control volume  $\Omega_{cv}$  around the vortical wake. Two surfaces contribute to the momentum in the  $\hat{\mathbf{z}}$  direction: the nozzle exit surface (red), and the time-varying material boundary between the wake and the external fluid (purple). The contributions to the momentum must balance so that the momentum flux through the nozzle exit surface is compensated by a mixture of control volume growth, motion, and pressure along the surface of the nearly-ellipsoidal control volume. (c) Pressure distribution around an ellipsoid moving steadily in the  $+\hat{\mathbf{z}}$  (downward) direction. Red coloring indicates positive pressures, blue indicates negative pressures, and white indicates zero pressure. Streamlines of the associated potential flow are drawn in black. To estimate the  $\hat{\mathbf{z}}$  - forces associated with this pressure distribution, I only integrate the pressure distribution directly below the nozzle (where the pressure is greater than 0 and the ellipsoid normal is primarily along  $\hat{\mathbf{z}}$ . 33

- 3-5 Different steps of the analysis process. (a) Calibration image of the nozzles. This image is used to determine the nozzle positions, their separation  $\Delta$ , their outer diameter in pixels  $d_{outer}$ , and the nozzle angle relative to the camera. (b) A typical frame of one nozzle after preprocessing and masking. The blue mask represents the CV identified using standard computer vision techniques. (c) Ellipsoid fit to the vortex bubble, which is used to estimate pressure. The semi-minor axis has length a and the semi-major axis has length b. While the vortex bubble does not perfectly fit the vortex structure, it serves as a good approximation to the shape in order to calculate the pressure along the leading edge of the CV.
- 3-6 Implementation of the volume calculation one frame of the control volume. The CV can be parametrized by the radius as a function of height  $\varrho(z)$ , and this value integrated from z = 0 to the maximum extent of the CV z = Z. Because each frame provides two measurements of  $\varrho(z)$  (to the left and right of the red centerline), I calculate the volume based on both measurements of  $\varrho(z)$  and average the two results to determine the volume of the CV in that frame. . . . . . . . . . . . .

39

42

49

- 4-1 Sequence of images from typical single-nozzle experiments. The top row shows the wake development for nozzle 1, the middle row shows wake development for nozzle 2. The bottom row shows a composite image for more direct comparison of the two wakes the left side of each frame is taken from nozzle 1, and the right side from nozzle 2. 48
- 4-2 Results of CV tracking for single nozzle experiments. Red lines represent the CV position and volume for nozzle 1, green lines represent the values for nozzle 2. Solid lines represent the median curve from 5 experiments, and the shaded regions represent one standard deviation. Both position and volume are roughly linear across the time range measured, with small sinusoidal oscillations arising from a shear instability along the jet. These oscillations are amplified in discrete time derivatives, but can be suppressed if the data is fit to a line.

4-5	Pressure and inertial contributions to thrust for single-nozzle. Because the vortex bubble growth is not linear, the pressure force grows non- linearly with time. When comparing the pressure forces to the inertial forces, observe that the scales on the vertical axes are different. The inertial forces are approximately ten times greater than the pressure forces at the times investigated	52
4-6	Thrust evolution with time for single-nozzle experiments. The thrust scale is dominated by the (constant) inertial contribution from the CV analysis.	52
4-7	Normalized average CV velocity for each experiment, as a function of nozzle spacing $\tilde{\Delta}$ . The CV velocity exhibits a strong and highly local dependence on $\tilde{\Delta}$ , increasing approximately as $1 - \tilde{\Delta}^{-6}$ . The blue squares represent data from nozzle 1, red triangles represent nozzle 2, and the black line represents a fitted 1-parameter model as described in the text.	53
4-8	Normalized average CV volume growth rate $\dot{V}_{cv}$ for each experiment, as a function of nozzle spacing $\tilde{\Delta}$ . $\dot{V}_{cv}$ remains nearly constant across all experiments, with the exception of some of the higher values of $\tilde{\Delta}$ . The blue squares represent data from nozzle 1, red triangles represent nozzle 2. The shaded region represents the standard deviation of $\dot{V}_{cv,\infty}$ measurements	53
4-9	Average vortex bubble growth rates for the minor and major radii, $\dot{a}$ and $\dot{b}$ . As two nozzles are brought into closer proximity, the growth rate $\dot{b}$ slows, and a concomitant increase in $\dot{a}$ is observed	55
4-10	Average vortex bubble velocity $\overline{w}$ . As the nozzles are brought together, the velocity decreases significantly, reflecting the trend $1 - C\widetilde{\Delta}^{-6}$ .	55
4-11	Normalized average inertial contribution to the thrust $T_{II}$ , as a function of nozzle spacing $\tilde{\Delta}$ . For close nozzle placement, the inertial force term drops by nearly 8% compared to the limiting value of a single nozzle. The outliers in $\dot{V}_{cv}$ are reflected in the calculation of $T_{II}$ .	56
4-12	Normalized average pressure contribution to the thrust $T_{III}$ , as a function of nozzle spacing $\tilde{\Delta}$ . As the nozzles are brought closer together, the average pressure force drops by nearly 30%. Despite this large fractional drop, because the pressure force is an order of magnitude smaller than the inertial force, the total thrust does not change as strongly.	56
4-13	Relative scale of the two contributors in the thrust calculation: pres- sure contribution to inertial component $(T_{III}/T_{II})$ . As was seen for the single nozzle experiments, the scale of the pressure contribution is much smaller than that of the inertial contribution, indicating that entrainment and CV acceleration is a more important process than the	r I
	ambient added mass reaction	57

4-14	Normalized average thrust over first 0.3s for each nozzle spacing tested. The normalized thrust should monotonically asymptote to 1 as $\tilde{\Delta} \rightarrow \infty$ . The solid line represents a fit to the 1-parameter model given by Eq. 4.2, which is discussed further in Section 5. In this case, $\overline{T}/\overline{T}_{\infty} = 1 - Co\tilde{\Delta}^{-6}$ for a value of the coupling coefficient, $Co = 2.42 \pm 0.11$ . The shaded region represents the standard deviation of the average thrust measured for the single nozzle ( $\tilde{\Delta} \rightarrow \infty$ ). Blue squares represent the mean values from nozzle 1, and red triangles represent the mean values from nozzle 2. The results of individual experiments are shown as colored circles. In many cases, these points are covered by the markers for the average.	59
5-1	(left) Estimated nozzle pressure $p_n$ as a function of nozzle spacing $\widetilde{\Delta}$ , calculated based on the geometric scaling described in Sec. 5.2. As opposed to the calculation of total thrust, which involved volume measurements and potential flow estimates, the values in this plot were based <i>solely</i> on the geometry and motion of the vortex wake. The nozzle pressure changes with $\widetilde{\Delta}$ as $p_n/p_{n,\infty} = 1 - C_p \widetilde{\Delta}^{-6}$ for a pressure coupling coefficient of $C_p = 2.77 \pm 0.03$ . (right) The scaling estimates were calculated for each moment throughout the experiment, and these time-series are shown for select values of $\widetilde{\Delta}$ . To arrive at the values plotted in the left plot, the average value of each time series was calculated.	68
5-2	<ul> <li>Images of salps (<i>Thalia Democratia</i>) in the ocean. (a) Solitary salp.</li> <li>(b) Chain of three salps. Images reproduced from Cifonauta database [50, 51].</li> </ul>	74
B-1	(a) Induction Geometry. (b) While the induced velocity is described by a complicated equation, for the values of $r$ that I am interested in (r/2a > 1.5), the induced velocity is well approximated by the scaling $(r/2a)^{-3}$	86
C-1	Time-series evolution of CV position and volume. This plot shows data calculated from the wake of nozzle 1 at a few values of $\widetilde{\Delta}$	89
C-2	Time-series evolution of vortex bubble geometry $(a, b)$ and growth rates $(\dot{a}, \dot{b})$ . Time derivatives in these plots are calculated numerically. This plot shows data calculated from the wake of nozzle 1 at a few values of $\tilde{\Delta}$ .	90
C-3	Non-normalized CV velocity, as a function of $\widetilde{\Delta}$ . Blue data points are calculated from wake measurements at nozzle 1. Red data points are calculated from nozzle 2.	91
C-4	Non-normalized vortex bubble geometry evolution (minor axis growth rate $\dot{a}$ and major axis growth rate $\dot{b}$ ), as a function of $\tilde{\Delta}$ . Blue data points are calculated from wake measurements at nozzle 1. Red data points are calculated from nozzle 2	91

C-5	Non-normalized average momentum and pressure contributions to thrust,	
	as a function of $\Delta$ . Blue data points are calculated from wake measure-	
	ments at nozzle 1. Red data points are calculated from nozzle 2	92
$\alpha$		

C-6	Time-series contributions to total thrust - pressure contribution (left)	
	and inertial contribution (right). This plot shows data calculated from	
	the wake of nozzle 1 at a few values of $\widetilde{\Delta}$	92

C-7 Thrust calculated from the wake of nozzle 1 at different values of  $\widetilde{\Delta}$ . 93

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# Chapter 1

# Introduction

One of the biggest challenges facing next generation underwater robots is efficient maneuvering at low speeds; vehicle capabilities such as advanced sensing and underwater rendezvous require precise position control and maneuverability. Such maneuverability, in turn, requires that thrusters that can deliver specific impulses rapidly and efficiently to the vehicle. Traditionally, underwater vehicles have been driven by propellers because of their high propulsive efficiency and speed in traditional marine transport operations. However, it is unclear that propellers can provide the necessary levels of thrust quantization, speed or efficiency required for precision maneuvers. By their design, propellers serve a different purpose than is called for in these delicate motion-control operations.

In these settings, pulsed jets are being increasingly used to augment vehicle maneuverability and improve efficiency at low-speeds. Pulsed jets offer many benefits over traditional propeller propulsion, including more precise impulse delivery, more rapid impulse delivery, and the ability to propel a vehicle using zero mass flux [1]. Pulsed jets also offer opportunities for swarm propulsion - pulsed jets are observed in nature for individual and group propulsion in species such as squid, jellyfish, siphonophores and salps. For salps and siphonophores uniquely, individual animals form chains where each member can independently control its jetting behavior. By synchronizing jet strength and timing, colonies of these animals can execute precise maneuvers and can reach high speeds efficiently [2–5]. Taking inspiration from nature, pulsed jets may prove to be an important technology for the development of scalable marine robotic swarms.

Before pulsed jets can be used more widely for underwater vehicles, certain design considerations must be addressed. To date, research efforts have focused on the thrust characteristics of single pulsed jets. However, the interaction of multiple pulsed jets is an important topic relevant to vehicle design and robot swarm control. For an underwater vehicle to be well-actuated and controlled, it will likely require multiple thrusters. The placement of these thrusters is a design decision that must account for the interactions between nearby thrusters. Similarly, in a swarm setting where multiple individual vehicles are ejecting jets, like is the case for salps, the wake dynamics and their interactions could have significant consequences for the thrust production and efficiency of the group. In these situations, it is necessary to understand how jet interactions will affect thrust production and efficiency for each of the jets.

In this thesis, I investigate this multi-jet interaction problem experimentally by visually observing the wakes formed by two jets of diameter D separated by a distance  $\Delta$ . I find that as the jets are brought together,  $\Delta \rightarrow D$ , the thrust T and efficiency  $\eta$ of each jet drop by nearly 10%. The thrust and efficiency fall off from the single-jet values,  $T_{\infty}$  and  $\eta_{\infty}$ , as

$$\frac{T(\Delta)}{T_{\infty}} = \frac{\eta(\Delta)}{\eta_{\infty}} = 1 - Co\left(\frac{D}{\Delta}\right)^{6},$$

where Co is a dimensionless "coupling number" that describes how strongly the twonozzle coupling affects thrust and efficiency. I develop scaling models to explain why the thrust and efficiency fall off as  $\Delta^{-6}$  based on the geometry of the problem and the pressure scales introduced by bringing two nozzles together. These models are used to generate predictions that demonstrate strong agreement with the measured data.

The data presented in this thesis also reveal interesting implications for the general dynamics of pulsed jet formation. My data contributes to a long-standing discussion on the source of nozzle over-pressure in pulsed jets. Existing interpretations suggest that pulsed jets generate more thrust than steady jets because of two factors: (1) added mass effects, or 'pushing off' of the fluid behind the jet, and (2) entrainment of ambient fluid into the forming vortex wake. While both of these factors certainly contribute to the nozzle over-pressure associated with pulsed jets, my data indicates that the entrainment of ambient fluid is responsible for over 85% of the measured thrust generated in the regime of Reynolds number  $Re \sim 350$ .

Finally, I discuss the implications of these results for underwater vehicle design and for the field of pulsed jet propulsion. The data and models suggests interesting possibilities for optimal behavior and even indicate the potential of thrust augmentation if the jets are controlled cleverly.

The thesis is organized into 6 chapters, and 3 appendices:

- Chapter 1 (this chapter) introduces the problem and thesis organization.
- In Chapter 2, I summarize background material relevant to pulsed jet propulsion and vortex interactions.
- In Chapter 3, I describe my experiment and the analysis framework used to visually estimate forces from the wake of a pulsed jet.
- In Chapter 4, I present the results of these experiments.
- In Chapter 5, I discuss the results, develop models that predict and explain the observed behavior, and discuss the implications of these models.
- In Chapter 6, I summarize the findings of this thesis and reiterate interesting directions for future study.

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## Chapter 2

# Background

### 2.1 Thrust generation with pulsed jets

#### 2.1.1 General Principles

Recent scientific interest in pulsed jet propulsion underwater dates back to 1977 when Weihs predicted that a continuous series of pulsed jets should provide more thrust than an equivalent steady jet [6]. In his argument, Weihs observed that pulsed jets roll up in to vortex rings that propagate downstream. Using the impulse associated with the developed vortex rings, he concluded that the time averaged thrust acting on the body can be higher than the thrust of an equivalent steady jet by a factor related to the added mass of the formed vortex rings. This insight underlies much of the understanding that has developed since - namely, ejecting a pulsed jet produces a vortex, which generates more thrust than a steady jet because of the higher impulse required to create the vortex ring.

Weihs's argument was developed further by Krueger and Gharib, who explained that the thrust boost associated with pulsed jets could be described by an increase in pressure at the jet's nozzle during early-time jet formation [7, 8]. Building off of earlier vortex ring research and the results of Gharib et al. [9], Krueger observed that, for a jet of diameter D with velocity  $u_j$ , the impulse I delivered by a pulsed jet can be broken up into two parts - an inertial term  $I_u$  traditionally associated with a steady jet, and an 'over-pressure' term  $I_p$ :

$$I = I_u + I_p \approx A_n t_p (\rho u_i^2 + p_n).$$

Here,  $A_n$  is the nozzle area,  $t_p$  is the duration of the jet pulse,  $\rho$  is the density of water,  $u_j$  is the jet velocity, and  $p_n$  is the 'nozzle over-pressure' associated with pulsed jet formation [7, 8]. When the pulse duration does not exceed than the critical vortex formation time  $t < T_f = 4D/u_j$ , he observed that the pressure term  $I_p$  contributes nearly half of the total impulse generated by the jet.

The origins of the over-pressure term can be traced to the complex dynamics associated with vortex ring roll up and vorticity shedding from the nozzle into the wake. For an intuitive understanding, consider that in a steady jet, the streamlines run parallel and straight out of the nozzle, indicating no normal pressure gradient  $\partial p/\partial n = 0$ . In this case, the pressure can be integrated radially from the jet out to infinity, demonstrating that the pressure everywhere in the jet is given by the freestream pressure  $p_0$ . If, however, the steady jet is replaced by the unsteady, pulsed jet, then the streamlines will curve around the ejected vorticity, reflecting the vortex rollup dynamics. In this case, the curved streamlines indicate a nonzero normal pressure gradient, suggesting that the nozzle pressure is not generally equal to the freestream pressure.

Explicitly, the relationship between flow curvature and nozzle overpressure was expressed by Krueger based on a scaling of the terms in the momentum equation [10]:

$$p_n \approx \rho \int_0^{D/2} u_z \frac{\partial u_r}{\partial z} \Big|_{z=0} dr$$

Here, D is the nozzle diameter, r is the radial coordinate,  $u_r$  is the radial velocity

field, and z is the axial coordinate with z = 0 representing the plane where fluid exits the nozzle.

An alternate explanation for nozzle over-pressure in pulsed jets has been derived based on the evolution of the vortical wake generated by the pulsed jet. Much like the original argument of Weihs, the added mass of the vortex structure can be used to describe the thrust benefit associated with nozzle over-pressure in a pulsed jet [7, 8, 11, 12]. Such descriptions of nozzle over-pressure typically build upon a control volume analysis to relate the nozzle over-pressure to forces required to generate the motion and growth of the vortex itself. Given the shape and velocity of a vortex ring, traditional added-mass calculations can be used to calculate the momentum flux that was required to set such a structure in motion. This way, force measurements in pulsed jets could be performed by measuring wake dynamics instead of measuring the flow-field near the nozzle [11].

#### 2.1.2 Factors that affect pulsed jet development

After the relationship was established between thrust augmentation in pulsed jets, nozzle over-pressure, and vortex wake dynamics, research grew on how different operating conditions and geometries alter the jet formation process and the associated nozzle over-pressure. Following traditional vortex dynamics literature, many of these studies focused on the production of circulation  $\Gamma$  under these different conditions, and how these conditions alter the critical time-scale for vortex formation  $T_f$  [13, 14]. Because of the relationship between  $\Gamma$  and  $p_n$ , changes in circulation also indicate concomitant changes in thrust production [10].

One set of these studies demonstrated how different ambient conditions affect vortex formation (and implicitly, the nozzle over-pressure and thrust production). Krueger et al. demonstrated that for a pulsed jet immersed in steady co-flow (background flow parallel to the jetting direction), the total circulation production decreases as the speed of the co-flow increases [13]. Then Dabiri and Gharib showed that in the case of counter-flow (background flow *anti*-parallel to the jetting direction), the vortex formation time was increased, and was associated with increased circulation in the vortex ring [14].

Another set of studies investigated how nozzle geometry and configuration alter the production of circulation and thrust in pulsed jets. These studies have indicated that circulation production can be controlled by changing, and even dynamically manipulating, the nozzle geometry [15–18]. Both Allen and Naitoh as well as Dabiri and Gharib independently looked at the effects of temporal variations in nozzle diameter during pulse ejection. These studies, which have relevance to biological propulsion schemes, indicate that closing the nozzle during jet ejection is associated with an increase in circulation, a decrease in vortex ring energy, and a decrease in mass transport away from the nozzle [15, 16]. When the nozzle is opened continuously during jet ejection, the vortex wake developed with a higher energy and no net change in circulation [11]. In a computational study, Rosenfeld et al. demonstrated that by using a nozzle with conical geometry instead of a straight tube, the rate that circulation is delivered to the external flow could be increased by 20% [17]. Whereas previous studies had considered axisymmetric nozzles, O'Farrell and Dabiri experimentally investigated the effect of using non-axisymmetric nozzles to eject a pulsed jet. Using elliptical and oval-shaped nozzles, O'Farrell and Dabiri identified changes in the arrangement of vorticity in the vortex ring cores, and also developed a framework to explain how circulation production changes with nozzle shape [18].

Many further studies have characterized the behavior and performance of *continuously* pulsed jets [7, 19, 20], which have been especially relevant for the development marine thrusters [1, 21–26]. In this setting, thrust production and propulsive efficiency have been characterized as functions of system geometry, jet velocity, and pulse frequency [22–24]. Additional studies have demonstrated the ability of multiple thrusters on a single vehicle to provide improved maneuverability for underwater vehicles [1, 21].

In order to directly compare pulsed jet performance with traditional steady propulsion vehicles, Ruiz et al. developed an efficiency metric based on the vortex wake behind a pulsed jet. Using a control volume approach, they related the thrust and energy usage of the vehicle to the momentum and energy associated with the wake's motion and growth. When considered in a quasi-steady limit (rapid, continuous pulsing of the jet), this new efficiency metric is similar to Freud or rocket efficiencies, but more appropriately accounts for the wake behind a pulsed jet. This new efficiency metric suggests that creating vortex wakes with higher added-mass can increase vehicle efficiency. Using experiments on a model vehicle, Ruiz et al. demonstrated that the vehicle efficiency was 70% greater when propelled by pulsed jets than when propelled by steady jets [25]. Then, in follow-up experiments, Whittlesey and Dabiri used the control volume analysis in conjunction with experimental data to relate the efficiency gains to vortex ring characteristics in the wake. This study demonstrated pulsing strategies that optimize vehicle efficiency [26].

Together, the results of experiments, design, and animal observation indicate how different factors – including nozzle geometry, ambient flow, and active control – can alter the vortex formation process during the development of a pulsed jet. As a result of this altered formation process, the production of circulation and thrust are modified from the behavior of an unperturbed circular jet. These previous results also reveal how propulsive quantities such as thrust and efficiency relate to the properties of the wake forming behind the pulsed jet. Finally, recent studies demonstrate how the relationship between propulsive quantities and wake dynamics can be exploited to calculate thrust and efficiency from measurements of wake dynamics.

#### Aside: vortex propulsion and animal locomotion

Because pulsed jets have been observed as a natural behavior in marine animals for quite some time, many recent developments in pulsed jet propulsion have been closely tied to those in animal locomotion. The most commonly known animals that exploit pulsed jets for locomotion include cuttlefish, squid [27–30], jellyfish [31– 35], siphonophores [5], sea squirts, pyrosomas, and salps [2–4]. The research tying pulsed jet theory into animal locomotion has focused largely on efficiency, describing techniques by which animals optimize behavior for different locomotion tasks. Of all of the interesting behaviors described in the references above, salps stand out because of how they cooperate for propulsion when individual individual jetting animals assemble into chains [2–4]. Since they are an interesting example of multi-jet locomotion, salp chains will be discussed in the context of my results in Chapter 5. For further details on the topic animal pulsed-jet locomotion, the reader is directed to the references listed above.

#### 2.2 Vortex ring interactions

Since the problem of pulsed jet propulsion is intimately related to that of vortex ring formation, it is likely that when two pulsed jets are brought together, their interactions will be governed by interactions vortex rings. The problem of interacting vortex rings is not new, having captivated the minds fluid dynamicists for well over a century. Recent studies have demonstrated the qualitative behavior of multiple vortex ring interactions [36–38]. Fohl and Turner forced two vortex rings to collide at wide range of incident angles ( $\theta_i = 10^\circ - 60^\circ$  separation), observing interactions that are strongly angle-dependent. When the angle is large enough ( $\theta_i > 30^\circ$ ), the collision results in a two-vortex structure propagating forward. Below the critical angle, the collision typically produces a single-vortex structure [36]. Oshima and Asaka investigated two-ring interactions in the limit  $\theta_i \to 0$ , corresponding to two vortices moving parallel to one another. Experiments in both air and water demonstrated that in this limit, three merging behaviors emerge, depending on the Reynolds number of the flow: (1) the two rings can merge and then separate again into two rings, (2) the two rings can merge into a single ring, shedding two small vortices as it advances, or (3) the two rings can merge into a single ring that oscillates as it advances [37, 38]. These qualitative behaviors were observed in my own early experiments as well.

Building off of the qualitative studies, later experiments provided insights into the mechanisms involved in the vortex ring interactions [39-41]. Oshima and Izutsu illustrated the importance of bridging and vorticity annihilation during the vortex ring merging. Using measurements of vorticity, they demonstrated that are were no instantaneous cut-and-reconnection events, but that instead the vortices connect by 'dissolving' and 'bridging' gradually. These results indicate that the vortex core thickness and curvature are important parameters that governed the merging process [39]. Lim compared these previous merging results to experiments of a vortex 'colliding' with its own image by ejecting a vortex ring towards an inclined wall. He observed that, as the vortex ring approaches the wall, significant secondary vorticity is produced and a shear layer along the wall is rolled up into a vortex structure that entangles part of the incident vortex ring. By contrast, his experiments with colliding rings did not demonstrate production of secondary vorticity and instead revealed strong mutual induction between the colliding rings along with significant vortex stretching and vorticity annihilation around the point where the vortices merge. These results indicate that the dynamics of two-ring interactions and connection are strongly governed by vorticity dynamics, not simple potential flow models [40].

Most recently, experiments on vortex topology have provided closer looks at vortex reconnection dynamics [41, 42]. Aref and Zawadzki used a topological framework to look at reconnection dynamics during vortex ring interactions. Based on simulations of rings interacting in different geometries, they illustrated that vorticity annihilation and local induction can lead to topological changes in the vortex structure, which are associated with different flow properties, such as helicity [41].

This result has interesting implications, suggesting the topological structures generated in a vortical wake will affect different wake dynamics, potentially providing different effects to propulsive or mixing properties. This idea can be further examined in the context of the experimental results of Scheeler et al., who closely tracked the core dynamics of vortex knots undergoing self-induced evolution. The topological evolution of the vortex knots revealed that a global flow property, helicity, is conserved in real fluids, illustrating how an understanding of reconnection dynamics can provide insight into the production of global flow properties when complex vortex structures are interacting [42]. It would be interesting to explore how topological descriptions of vortex dynamics during pulsed jet formation can be exploited to design or optimize thruster geometry for specific behavior.

So far, studies on vortex interactions have remained focused on times well beyond initial vortex formation. It would be instructive and beneficial to extend some of these studies to earlier times during formation, in order to better describe the vortex formation process in more complex geometries.

# Chapter 3

# **Experimental Framework**

In this chapter, I describe the experiment I designed to test the effect of nozzle spacing on thrust production in a two-jet system. One point to keep in mind is that the ultimate goal of these experiments is the relative behavior of 2-jet systems as the spacing  $\Delta$  is varied. So although some of the approximations made in the analysis may introduce systematic offsets from the true forces generated by the jets, I expect this shift to be the same for all of the experiments so that the measured behavior still robustly reflects the dependence of thrust on nozzle spacing.

### 3.1 Experimental apparatus

In order to understand the evolution of thrust and efficiency in a two-nozzle pulsed jet, I imaged pulsed jet formation at early times, using the motion of fluorescent dye in the wake to estimate hydrodynamic forces. The jets were created in a  $(30.4 \text{cm})^3$  cubic water tank using two D = 6.35 mm inner-diameter stainless steel nozzles. The nozzles were purchased with a slightly smaller bore which was machined out to the final diameter. The outer diameter of the nozzles was 9.52mm, and the faces of the nozzles were machined smooth and deburred, but were not tapered to a fine angle or sharp edge as has been done in other experiments [8, 19, 43].



Figure 3-1: Experimental setup. (a) Experiment schematic depicting all of the system components. The side view reveals the illumination and imaging system, which images the jets from the front. (b) Image of backlit nozzles revealing the field of view and geometry. (c) Typical fluorescence image of the pulsed jet wake during an experiment.

The nozzles were mounted on a linear rail so that nozzle spacing could be varied continuously. Flow through each nozzle was driven by an independent pressure reservoir. For these experiments, the pressure reservoir was hydrostatic, consisting of two 60mL syringes filled to capacity. During a typical experiment, the height of the fluid in the reservoirs changed by less than 0.5mm, allowing me to ignore the height changes in the hydrostatic pressure calculation over the course of an experiment. The reservoirs were elevated L = 570mm above the water surface for an expected pressure head of  $p_h = \rho g L \approx 5500$ Pa. However, as described below, losses between the reservoir and the nozzle reduced the pressure head by an order of magnitude. The reservoir was connected to the nozzle with 6.35mm Tygon tubing.

Between the reservoir and nozzles, additional hardware measured and controlled the flow in the experiment. Volume flux was measured using a low-inertia, positivedisplacement flow meter (FCH-m-POM 97478039; BIO-TECH e.k.) placed inline between the reservoir and the nozzle. Typical flow measurements are shown in Fig. 3-2. An inline solenoid valve (CNYUXI 2W-025-08) controlled the flow. When open, the valve constricted fluid flow to a 2mm diameter, setting the typical jet velocity  $u_i$  to  $\sim$ 50mm/s for a 0.3s pulse. Using the nozzle diameter as a length scale, the typical jet Reynolds number is  $Re \approx 350$ .

A schematic of the experimental setup is provided in Fig. 3-1. In this geometry, the separation between the tank walls and the nozzle was typically 120mm, and the nozzle exit plane was submerged 40mm below the free surface. With this configuration, edge-effects should be negligible during the vortex formation, and the hydrostatic pressure at the nozzle exit is  $p_{n,g} = \rho g(40\text{mm}) = 392\text{Pa}$ . Because of the pressure loss through the tubing and valve, the effective driving pressure was estimated using a simplified momentum equation for unsteady pipe flow:

$$p_d - p_{n,g} \approx \frac{\rho u_j L}{t} = 114 \text{Pa.}$$

Fig. 3-2 shows the volume flow Q through the nozzles, measured by the flow meters. The flow meters are inline, positive-displacement sensors, so the initial reading  $Q_0$  is not synchronized between the two sensors. The discrepancy in the initial state results in the horizontal offset visible in the figure. Although the flow meters are



Figure 3-2: Volume flux through the nozzle as measured by the flow meters attached to each reservoir. The volume flow through the nozzles is approximately linear in time, so to calculate the volume flow rate, the data for each nozzle are fit to a line. Here, the volume flow rate through each nozzle is  $\dot{Q} = 1.68 \pm 0.04$  mL/s.

read at a frequency of 2kHz, the sensors do not register flow continuously. Instead, they transmit pulses when a minimum volume has passed through them. Because of this minimum volume constraint on the sensor, and the low flow rates in this experiment, there are relatively few data points to define a flow profile. However, the points accurately reflect the volume flux, and because of their linearity over the range of the experiment, the volume flow rate  $\dot{Q}$  is calculated using a linear fit, as shown in the figure. In this figure, the fit yields the same volume flow rate for each nozzle, of  $\dot{Q} = 1.68 \pm 0.04$  mL/s. Given the nozzle area, this flow rate corresponds to a jet velocity of 53.20 mm/s. Across all of the experiments at different nozzle spacings, I did not observe any significant deviation in the jet velocities. All jet velocities fell within the range 48.2-56.4 mm/s.

In order to follow the evolution of the developing jets, the reservoirs contained water mixed with fluorescein dye  $(5 \times 10^{-7} \text{ M} \text{ fluorescein sodium salt in water})$ . This way, all of the fluid ejected from the nozzle was marked with a fluorescent tracer, while the fluid in the tank was transparent. The jets were illuminated with a blue (462nm) laser diode module (1.5W optical power; Lasertack LDM-462-1400-C). The laser module emits a 4mm-diameter gaussian beam that was expanded into a 4mm thick laser sheet using a cylindrical lens. This laser sheet was centered on the nozzles to illuminate the central plane of both jets, as depicted in Fig. 3-1.

When exposed to the blue laser sheet, fluorescein dye emits green light (532nm), which was recorded at 400 frames/second using a high-speed camera (Phantom Miro 320s; Vision Research Inc.). This frame rate corresponds to a time resolution of 2.5ms.

To prevent scattered laser light from affecting the optical signal, a 495nm highpass optical filter (Thorlabs FGL495S) was placed in front of the camera as shown in Fig. 3-1a.

The hardware was digitally controlled with a custom Python script that synchro-

nized the values, laser, and camera acquisition. The same script also read the flow meter values continuously throughout the experiment. The computer interfaces with the hardware using a high-accuracy data acquisition board (LabJack U6 Pro; LabJack Corporation). Triggers were sent as +5V TTL pulses.

Each experiment lasted for 1 second. First, the camera and laser were triggered, then the valves were opened providing a sudden pressure gradient to initiate the pipe flow. After 0.3s, the valves were closed, and the flow was recorded until 1s had passed from the start of the experiment. Finally, the tank was allowed to settle for 3 minutes before the next experiment is performed. This ensured that no residual vorticity or dye would impact the measurements of consecutive experiments.

Fig. 3-1c shows a typical image taken during pulsed jet development for the twonozzle configuration. A series of images spanning the whole pulse period is shown in Fig. 3-3. When initially ejected, the unsteady and localized flow generates circulation at the nozzle edges, creating a vortex ring that dominates the dynamics at early times during jet ejection. The vortex ring is continuously fed momentum, circulation and energy by the nozzle, and maintains an ellipsoidal structure at early times, when the interactions are weaker. For the two-nozzle experiments, as the rings continued to grow and interact more strongly, they deformed significantly from their ellipsoidal structure, and evolved into more complex three-dimensional vortex structures.



**Figure 3-3:** A sequence of images at different points in the jet's early development. These frames correspond to a single jet. Not only does the center of massmove as more fluid is ejected, but the volume grows and the wake's shape evolves with time. These frames are only a subset of a typical experiment; for the full analysis, video data is acquired every 2.5ms.

#### **3.2** Derivation of thrust estimation technique

The thrust produced by an underwater jet can be calculated by measuring the momentum flux through the nozzle. Surrounding the jet by a control volume (see Fig. 3-4a for depiction of the control volume around a conceptual jet thruster), the forward-thrust  $T(-\hat{\mathbf{z}})$  must be balanced by the inertial momentum transfer out of the nozzle and the pressure on the nozzle exit plane. Assuming that the jet velocity and pressure are roughly constant along the nozzle exit plane, I can integrate the momentum equation around the thruster to recover an equation for the thrust produced by the jet:

$$T = \int_{S_n} \left[ \rho(\mathbf{u} \cdot \hat{\mathbf{z}})^2 + p_n - p_0 \right] dA$$
  

$$\approx A_n (\rho u_j^2 + p_n - p_0), \qquad (3.1)$$

where  $A_n$  is the nozzle area,  $\rho$  is the fluid density,  $u_j$  is the fluid velocity exiting the nozzle,  $p_n$  is the pressure along the nozzle-exit plane, and  $p_0$  is the free-stream pressure. In a steady jet, because the streamlines exiting the nozzle are straight, there is no pressure gradient normal to the streamlines, and the the nozzle pressure is identically the free-stream pressure so that  $p_n - p_0 = 0$ . However, because a pulsed jet initially rolls up into a vortex ring, for early times the nozzle pressure is not equal to the free-stream pressure, and this 'nozzle over-pressure' contributes significantly to thrust production [7, 8].

Ideally, I would like to directly measure the thrust produced by the nozzles. However, because the forces are so small (~  $10\mu$ N in my operating regime), this is not a viable approach with existing load sensors. Another option would be to measure the different components of the thrust (jet velocity and nozzle pressure). However, there are few robust methods to non-invasively measure the pressure field in front of the nozzle, which is expected to contribute significantly to the thrust profile. Lacking direct measurement options, I resort to an indirect method that allows me to estimate the thrust produced by the jets.

This indirect approach can be derived analytically by using a second control volume,  $\Omega_{cv}$ , around the vortical wake (Figs. 3-4a and 3-4b schematically depict the control volume geometry. A similar approach is taken by Ruiz et al. [25]). This control volume encloses all of the fluid ejected from the nozzle, as well as any fluid entrained into the vortex structures that form. Given this geometry, the leading region within the control volume has been referred to as a "vortex bubble" [7, 8, 25, 43, 44]. The front of the vortex bubble is coincident with the material surface between fluid



Figure 3-4: Estimating thrust based on wake evolution. (a) The control volumes around a model vehicle and its vortical wake. The thrust experienced by the vehicle is balanced by the rate of momentum change in the wake. (b) A detailed control volume  $\Omega_{cv}$  around the vortical wake. Two surfaces contribute to the momentum in the  $\hat{\mathbf{z}}$  direction: the nozzle exit surface (red), and the time-varying material boundary between the wake and the external fluid (purple). The contributions to the momentum must balance so that the momentum flux through the nozzle exit surface is compensated by a mixture of control volume growth, motion, and pressure along the surface of the nearly-ellipsoidal control volume. (c) Pressure distribution around an ellipsoid moving steadily in the  $+\hat{\mathbf{z}}$  (downward) direction. Red coloring indicates positive pressures, blue indicates negative pressures, and white indicates zero pressure. Streamlines of the associated potential flow are drawn in black. To estimate the  $\hat{\mathbf{z}}$ - forces associated with this pressure distribution, I only integrate the pressure distribution directly below the nozzle (where the pressure is greater than 0 and the ellipsoid normal is primarily along  $\hat{\mathbf{z}}$ .

that was expelled from the nozzle, and fluid that started external to the jet. In experiments, this material surface is easily visualized when dyed fluid is ejected into a clear fluid, as shown in Figs. 3-1c and 3-3. Vortex roll-up during early jet formation causes this material surface to resemble an oblate ellipsoid of revolution, enclosing the forming vortex ring and most of the vorticity in the flow (some of the vorticity diffuses beyond the extent of the material surface described above [43, 44]).

To calculate thrust given this control volume, consider the vertical  $(\hat{\mathbf{z}})$  momentum conservation equation:

$$\frac{\partial}{\partial t} \int_{\Omega_{cv}} \rho \mathbf{u} \cdot \hat{\mathbf{z}} \, dV + \rho \int_{S_{vb}} (\mathbf{u} \cdot \hat{\mathbf{z}}) \mathbf{u}_{rel} \cdot \hat{\mathbf{n}} \, dA + \int_{S_{vb}} (p_{vb} + p_0) \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \, dA$$
term 1 - unsteady flow
$$- \int_{S_n} \rho u_j^2 + p_n \, dA = 0 \quad (3.2)$$
term 4 - momentum flux from nozzle

Here, the domain  $\Omega_{cv}$  represents the the entire control volume as highlighted in Fig. 3-4b,  $S_{vb}$  represents the control surface bounding the (roughly) ellipsoidal vortex bubble,  $S_n$  represents the control surface at the nozzle exit plane,  $\hat{\mathbf{n}}$  is the unit normal at each point on the control surface,  $\mathbf{u}$  is the local velocity at each point, and  $p_{vb}$  is the pressure on the surface of the vortex bubble.

To simplify Eq. 3.2, I make use of several empirical observations and assumptions. Focusing first on the first term, I rewrite the integral in terms of the CV volume and average CV velocity,

$$\frac{\partial}{\partial t} \int_{\Omega_{cv}} \rho \mathbf{u} \cdot \hat{\mathbf{z}} \, dV = \frac{\partial}{\partial t} (\rho \overline{u}_{cv} V_{cv}).$$

 $\overline{u}_{cv}$  is the average velocity around the entire control volume. Given the axial symmetry of the problem, this velocity should be purely in the  $\hat{z}$ -direction. While this equation is exact, without a measurement of u at each point within the CV, it cannot be used.

However, the average velocity  $\overline{u}_{cv}$  can be approximated as the average z-velocity of the control volume center of mass,  $\dot{z}_{cm}$ .

Second, observe that on most of the control surface  $S_{vb}$ , the surface evolves with the fluid so that there is no velocity flux, and  $\mathbf{u} \cdot \hat{\mathbf{n}} = 0$  on  $S_{vb}$ . Over the region of  $S_{vb}$  where entrained fluid enters the control volume, I assume that the entrainment is nearly tangential so that even in this region,  $\mathbf{u} \cdot \hat{\mathbf{n}} \approx 0$ . This assumption is consistent with empirical observations in similar experiments [25, 44]. Taken together, these observations indicate that the second term of Eq. 3.2 is negligible.

Third, I separate the two pressures in the third term and note that integrating the free-stream pressure around this control surface will cancel everywhere except for directly below the nozzle. This allows the integral to be rewritten as

$$\int_{S_{vb}} p_0 \mathbf{\hat{n}} \cdot \mathbf{\hat{z}} \ dA = \int_{S_n} p_0 \ dA,$$

which can be combined with the fourth term of Eq. 3.2. This final step reveals the modified fourth term as the thrust that is given in Eq. 3.1.

Leveraging these observations, and rearranging terms in Eq. 3.2, the thrust produced by the jet can be expressed in terms of the dynamics in the jet wake:

$$T = A_n \rho u_j^2 + A_n p_n$$

$$\approx \rho \overline{\dot{u}}_{cv} V_{cv} + \rho \overline{u}_{cv} \dot{V}_{cv} + \underbrace{\int_{S_{vb}} p_{vb} \, \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \, dA}_{\text{III}}.$$
(3.3)

A similar control volume approach has been used previously as an analytical approach to understand energy and thrust production in single pulsed-jet systems. Krueger used such a model to relate his observations of thrust production to an added mass effect and an additional pressure present at the nozzle exit plane [7, 8]. Later, Dabiri expanded on the idea of using wake measurements to estimate swimming forces [11, 12], and Ruiz et al. used a similar model to estimate the efficiency of a continuously-pulsed jet [25].

The present formulation for jet thrust differs from previous formulations because it relies on different measurable quantities. In previous formulations, momentum equations have been cast into a form that allowed thrust to be calculated from vorticity or velocity measurements. Building off of a different approach, I derived the thrust formula in terms of the developing vortex bubble, which can be tracked using fluorescent dye. For early times during vortex formation, it is experimentally easier to track a region of fluorescent dye than it is to track particle motion near a nozzle exit. I sought this representation to allow for more robust estimates at early times, which is the operating regime relevant to motion- and orientation-controlling impulses in an underwater vehicle.

Momentum conservation on the control volume (CV) surrounding the jet wake (Eq. 3.3) reveals that the thrust generated by the pulsed jet can be broken down into three independent measurable quantities. Term I represents the force to instantaneously accelerate the CV the  $\hat{\mathbf{z}}$  direction. Term II represents the force required to add mass to the growing CV, both by injection and entrainment. Both of these terms can be approximated by tracking the CV volume and center of mass position at each time during its growth. Term III describes the pressure around the growing vortex bubble that resists its growth. This term can be thought of in terms of the added mass associated with the growing vortex bubble as it pushes all of the external fluid out of the way [7, 11]. Term III requires knowledge of the pressure field surrounding the vortex bubble, which is not directly measurable from the fluorescence experiments.

In order to circumvent the need to directly measure pressure on the vortex bubble, I estimate the pressure on the vortex bubble using the common approximation that flow outside the vortex bubble is irrotational. Although I expect the vorticity inside
the vortex to diffuse beyond the extent of the fluorescein dye [44], for short times, the pressure variations from local diffused vorticity should be small compared to the pressures associated with the added mass of the vortex bubble motion.

By assuming flow around the vortex bubble is inviscid, and leveraging the assumption that the vortex bubble is approximately ellipsoidal (validity for early times demonstrated *a posteriori*), the pressure on the leading surface of  $S_{vb}$  can be equated to to the pressure experienced by an ellipsoid that is translating and expanding unsteadily in a potential flow. See Fig. 3-4c for an example of this pressure field.

This potential flow model reduces the pressure term (Eq. 3.3-III) to a function of vortex bubble motion and geometry, which allows the total thrust (all three terms of Eq. 3.3) to be estimated entirely using measuring the motion and growth of the fluorescent jet wake.

#### 3.3 Measurements & processing

Based on the analytical description of the thrust production process, the thrust estimation process can be broken up into 7 steps once a video of the experiment is acquired:

- 1. Analyze a static image of the experiment (calibration image) for nozzle positions, orientation, and size.
- 2. Preprocess video frames to improve image processing results.
- 3. Identify control volume in video frames (Fig. 3-5b).
- Calculate the size and position of the ellipse that bounds the vortex bubble (Fig. 3-5c).
- 5. Calculate inertial component of CV momentum by tracking CV volume and center of mass in each frame.

- 6. Calculate pressure along leading edge of the CV by using vortex bubble motion and growth. Calculate the associated pressure force by integrating the pressure on the section of the surface directly below the nozzle.
- 7. Calculate time-varying thrust during nozzle formation using the momentum equation (Eq. 3.3). For the calculation, use measurements of CV kinematics from step 5 and add the integrated pressure from step 6.

Using this approach, I analyze the high-speed fluorescence videos of the jet formation and development. Jets are ejected for a duration of 1s, but the jet development is only tracked for the first 0.3s while the jets develop. This is equivalent to calculating the properties of a jet with stroke ratio of  $L/D = u_j t/D \leq 2$ . Such small stroke ratios reflect the desired operating regime for short pulses to be used in vehicle maneuvering. Additionally, during this short time window, the vortex wakes retain their ellipsoidal structure, satisfying the assumptions of the analysis technique.

Although this technique relies on many approximations to estimate the force, the ultimate goal of this project is to compare the thrust produced by identical jets under modified operating conditions (proximity to a second jet). The approximations made in this technique should systematically impart any inaccuracies to all of the measurements, making the measurement of relative forces fairly robust. However, as a result of the approximations, this technique may prove less reliable if it is applied to problems with more complex geometry, or if the goal of the analysis is an exact force measurement.

Finally, it should be noted that while this control volume approach provides a means by which to estimate the thrust being generated by the jet, it does not directly indicate what kinds of interactions cause the observed behaviors. The values calculated with this method only provide a proxy for the forces. Additional modeling or experiments are required to determine the physical mechanism producing the observed effects. This point is discussed further in Chapter 4.

#### Image Calibration (Step 1)

Before acquiring fluorescence data, I photograph the experimental field of view under backlit conditions (see Fig. 3-5a). This calibration image provides the nozzle locations and magnification (px/mm) for each experiment, so that experiment images can be analyzed more robustly, and analysis results can be converted to physical units.

This analysis step provides the location of the nozzle centerline, which is used as the symmetry axis for later computations of CV volume and radius.

At this point, the orientation  $\theta_n$  of the nozzles relative to the vertical is also measured, so that images can be rotated and aligned with the global analysis coordinate system before processing.



Figure 3-5: Different steps of the analysis process. (a) Calibration image of the nozzles. This image is used to determine the nozzle positions, their separation  $\Delta$ , their outer diameter in pixels  $d_{outer}$ , and the nozzle angle relative to the camera. (b) A typical frame of one nozzle after preprocessing and masking. The blue mask represents the CV identified using standard computer vision techniques. (c) Ellipsoid fit to the vortex bubble, which is used to estimate pressure. The semi-minor axis has length a and the semi-major axis has length b. While the vortex bubble does not perfectly fit the vortex structure, it serves as a good approximation to the shape in order to calculate the pressure along the leading edge of the CV.

#### Image Analysis (Steps 2-4)

The goal of the image processing is twofold: (1) identify the material boundary for control volume analysis, and (2) track the ellipsoidal vortex bubble needed to estimate the pressures on the control surface.

In order to isolate the fluorescent wake, I use standard image processing techniques implemented in the Python programming language [45, 46]. It is easiest to analyze each wake independently, so for experiments with two nozzles, the images of each wake are automatically cropped to only bound the relevant wake, and the two nozzles are analyzed independently.

For each frame of a fluorescent video sequence, image contrast is enhanced with a local contrast filter, and the control volume is then segmented using a local Otsu threshold [46, 47]. The thresholding results are improved by using knowledge of the nozzle positions. The results of the thresholding operation are then filtered using a horizontal hole-filling algorithm. This operation fills any gaps in the segmentation from noise or entrained fluid, generating the mask shown in transparent blue in Fig. 3-5b. Using this filled region, the boundary is calculated using standard edge-finding techniques [48]. At this point, the masked region and boundary represent a 2D-slice of the axisymmetric control volume and control surface, respectively. By assuming that the control volume is axisymmetric, all calculations over the entire volume can be reduced to calculations on this 2D slice.

Finally, the position and size of the ellipsoidal vortex bubble are calculated. To do this, the maximum diameter of the CV is identified, and taken to represent the major axis of the vortex ellipsoid, with length 2b. The point where the major axis intersects the symmetry axis (identified in the image calibration step; see solid lines in Fig. 3-5a) is chosen as the ellipsoid center. The ellipsoid semi-minor axis, of length a, is determined as the vertical distance from the ellipsoid center to the leading edge of the vortex bubble. This technique yields an ellipsoid fit like the one shown in Fig. 3-5c. Tracking these vortex bubble properties for each frame produces time series for a and b as well as the vortex bubble's z-position  $z_{vb}$ . Such time series are presented in the following chapter in Figs. 4-3 and 4-4.

#### Inertial contribution to momentum (Step 5)

Once the control volume is identified in the image analysis step, the inertial terms in the thrust equation (Eq. 3.3-I and II) can be calculated directly by measuring the CV center of mass and volume.

Because the control volume is assumed to be axisymmetric, its center of mass can be calculated from the 2D mask identified in the previous step. A standard center of mass algorithm [45] is applied to the binary mask in order to identify the axial  $(\hat{\mathbf{z}})$ and radial  $(\hat{\mathbf{r}})$  position of the control volume center of mass.

To calculate CV volume from these 2D slices, the boundary of the 2D mask is assumed to be axisymmetric about the (vertical)  $\hat{z}$  axis. Because the mask is filled in with a horizontal hole-filling algorithm, the radial extent of the mask,  $\rho$ , is single valued as a function of vertical height, z. Therefore, given these two observations, the volume can be calculated by integrating radial extent of the boundary, in cylindrical coordinates, along its height (to Z):

$$V_{cv} = \int_0^Z \pi \varrho(z)^2 dz.$$

Using the image data, this integral is calculated with a trapezoidal integration routine. Since each frame yields two independent measurements of the control surface's radial extent (left and right sides of the symmetry axis - see Fig. 3-6), the mask boundary can be divided in half and the CV volume  $V_{cv}$  calculated for on each half independently. The two volume calculations are then averaged to provide a volume measurement for the control volume in that frame.

By calculating control volume position  $z_{cv}$  and volume  $V_{cv}$  in each frame, I can measure the time evolution of these quantities. These time series allow me to calculate  $\overline{u}_{cv} \approx \dot{\overline{z}}_{cv}$  and subsequent temporal derivatives that are necessary for the calculation in the first two terms of Eq. 3.3.



**Figure 3-6:** Implementation of the volume calculation one frame of the control volume. The CV can be parametrized by the radius as a function of height  $\rho(z)$ , and this value integrated from z = 0 to the maximum extent of the CV z = Z. Because each frame provides two measurements of  $\rho(z)$  (to the left and right of the red centerline), I calculate the volume based on both measurements of  $\rho(z)$  and average the two results to determine the volume of the CV in that frame.

#### Control surface pressure estimation (Step 6)

In order to compute the last term of the thrust equation (Eq. 3.3-III), I estimate the pressure on the leading edge of the control surface. This restricted portion of the control surface is used because the most significant pressure contribution to vertical  $(\hat{z})$  momentum balance will come from the pressure on this face.

To estimate the pressure on the control surface, I evaluate the velocity potential  $\phi$  around a translating oblate ellipsoid of revolution. Using the Bernoulli equation:

$$p_{vb} - p_0 = \frac{\rho}{2} (\nabla \phi)^2 \Big|_{S_{vb}} + \rho \frac{\partial \phi}{\partial t} \Big|_{S_{vb}}.$$
(3.4)

As shown in Appendix A, the velocity potential can be written in terms of ellipsoid geometry (principal axes a along  $\hat{\mathbf{z}}$  and b along  $\hat{\mathbf{r}}$ ) and velocity ( $w\hat{\mathbf{z}}$ ), which are measured as a function of time in step 4. Time derivatives are calculated with using central differences, and the results are smoothed by a gaussian window with  $\sigma = 2$ data points. Because the pressure calculation only needs to be evaluated on the ellipsoid surface, the spatial derivatives can be calculated analytically as shown in Appendix A, and the pressure around the ellipsoid can be calculated explicitly as a function of a, b, and w for each frame. An example of the pressure distribution around the ellipsoid is shown in Fig. 3-4c for an oblate ellipsoid translating steadily in the  $+\hat{z}$  direction.

Finally, to calculate the pressure contribution to the thrust equation, I integrate the pressure acting on the control surface. Because the pressure is highest directly below the nozzle, as is the  $\hat{\mathbf{z}}$  component of the control surface normal  $\hat{\mathbf{n}}$ , the pressure is only integrated directly below the nozzle. The integral is calculated numerically using trapezoidal integration, and accounting for the radius in the axisymmetric integral. Because this integral is not calculated across the whole surface, where the pressure can become negative, it will tend to slightly overestimate the total pressure force experienced by the control volume. The purpose of this calculation is to estimate the pressure on the leading control surface, which is associated with the added mass effects of the jet pushing ambient fluid out of the way.

#### Thrust calculation (Step 7)

Once the inertial and pressure terms of the momentum equation (Eq. 3.3) are calculated, they can be added together to provide an estimate for the total thrust produced by the pulsed jets. Since each calculation was performed frame-by-frame, an entire video provides the time evolution of the thrust force, as shown in Fig. 4-6.

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## Chapter 4

### **Experiment Results**

Using the experimental methods described in Chapter 3, I track the evolution of jet wakes for individual jets  $(\Delta \to \infty)$ , and for select nozzle spacings increasing from  $\Delta = 1.5D$ . Because  $\widetilde{\Delta} = \Delta/D$  arises as a natural dimensionless group in this problem, I will generally use  $\widetilde{\Delta}$  instead of  $\Delta$ .

The first half of this chapter presents data describing the behavior of the thrust production for the two nozzles used in my experiments. This section compares their behaviors when they eject jets individually,  $\Delta \to \infty$ .

The second half of this chapter investigates the interactions between two nearby, parallel jets and demonstrates the effects of jet interaction on thrust production. Using the available data, the relative importance different contributors to thrust production can be identified, as can their scalings with  $\tilde{\Delta}$ .

Supplemental video data can be found in the MIT library archives with the physical thesis, or online (as of Jan 27, 2016) at:

http://www.youtube.com/playlist?list=PLZz0i65P08GYvkzeKpQF5zCYFk0gp9B1J

#### **Description of experiments**

Experiments were performed with two nozzles at  $\widetilde{\Delta} = 1.74, 1.77, 1.94, 2.15, 3.30, 3.80$ , and for each of the two nozzles individually, which corresponds to the limit  $\widetilde{\Delta} \to \infty$ . For each configuration of  $\widetilde{\Delta}$ , 5 experiments were run. Each experiment was analyzed independently, and the results are averaged together in the plots presented in this section. Data points typically represent the median value from the 5 experiments, and the error bars represent the standard deviation. In some plots, the large data point represents the median, and the values from individual experiments are plotted as smaller markers.

For each experiment, I analyzed each of the two jet wakes independently, according to the assumptions laid out in the previous chapter. This provided me with two measurements of thrust generation for each experiment performed. In some experiments ( $\tilde{\Delta} = 1.74, 1.77, \text{nozzle 2}$ ) the wake deformed significantly beyond what could be treated as a quasi-axisymmetric ellipsoid, so the CV could not be reliably fit to an ellipsoid at later times, although CV average measurements were unaffected. The data relying on ellipse fits for these runs was ignored. For all experiments, the wake is analyzed between t = 0.08 ms and t = 0.30 ms. Because the initial startup flow is slow, analysis of the frames before 0.08 ms did not generate consistent results. The time t = 0 corresponds to the moment when the solenoid valves are opened and the jets begin to develop.

#### 4.1 Analysis of single-jet thrust production

Fig. 4-1 shows an image series of typical jet development for each of the two nozzles when operating independently. From these images it is clear that there are slight experimental differences between the two nozzles. The wake from nozzle 2 appears to advance slightly more slowly than that of nozzle 1, and the shape of the wakes differ slightly. Most noticeably, the vortex spiral is more elongated (vertically) in the wake of nozzle 2 than that of nozzle 1. An observation that is more easily noticed in the videos is that the two wakes experience a shear instability causing the straight jetting region to develop an oscillating profile. Although both nozzles experience a similar instability, the onset of the instability differs between the two nozzles (even when the jets form independently), introducing a phase delay between the two instabilities. In the time-series plots shown below, this behavior manifests as sinusoidal oscillations in the measured quantities. Values for nozzle 1 and nozzle 2 typically oscillate around the same mean, but with different phase. While these oscillations will be apparent in time-series plots, for the final plots of average thrust produced, these effects are averaged out.

To differentiate the two nozzles in the following plots, I use the notation  $\widetilde{\Delta} \to \infty_1$ to denote nozzle 1, and  $\widetilde{\Delta} \to \infty_2$  for nozzle 2.

Fig. 4-2 shows the time-evolution of CV position and volume. These quantities are used to calculate the inertial contribution to thrust, as described by Eq. 3.3 (terms I and II). Given the linear tendency of this data, perturbed by small sinusoidal oscillations, it is reasonable to ignore the oscillations and only consider the average behavior by fitting a line to the position and volume.

This simplification has two important consequences for the interpretation of the total thrust. First, if the CV position evolves linearly in time, then the average CV velocity  $\bar{u}_{cv}$  is a constant, meaning that the first term of Eq. 3.3,  $T_I = \rho \dot{\bar{u}}_{cv} V_{cv} = 0$  since  $\dot{\bar{u}}_{cv} = 0$ . Hence, the early-time relationship between thrust and wake inertia is dominated by the growth of the CV (both from jet ejection and from ambient entrainment). Second, since the linearization yields  $u_{cv} = const$  and  $\dot{V}_{cv} = const$ , the total inertial contribution to thrust will be a constant for the duration of the experiment. This contribution is plotted in Fig. 4-5. Ultimately, since I am interested

in the leading-order effects of nozzle separation on thrust production, the remainder of the analysis will be performed with the linearized data.

In addition to the CV growth, the growth and motion of the vortex bubble are tracked. The evolution of the major and minor radii, b and a respectively, is plotted in Fig. 4-3. Since these quantities oscillate more dramatically, they are not fit to lines, and their numerical derivatives are used to calculate the leading edge pressure, as described in Sec. 3.3. The vortex bubble, however, does follow a fairly linear trajectory as it moves, as is shown in Fig. 4-4. This path is fit to a line, yielding a single value for the vortex bubble velocity w that is used in the pressure calculation. After it is calculated, the pressure is integrated to provide the vortex bubble pressure contribution to the total thrust. The pressure contribution is shown alongside the inertial contribution in Fig. 4-5.



**Figure 4-1:** Sequence of images from typical single-nozzle experiments. The top row shows the wake development for nozzle 1, the middle row shows wake development for nozzle 2. The bottom row shows a composite image for more direct comparison of the two wakes - the left side of each frame is taken from nozzle 1, and the right side from nozzle 2.



**Figure 4-2:** Results of CV tracking for single nozzle experiments. Red lines represent the CV position and volume for nozzle 1, green lines represent the values for nozzle 2. Solid lines represent the median curve from 5 experiments, and the shaded regions represent one standard deviation. Both position and volume are roughly linear across the time range measured, with small sinusoidal oscillations arising from a shear instability along the jet. These oscillations are amplified in discrete time derivatives, but can be suppressed if the data is fit to a line.

One fact that becomes immediately apparent when comparing these two contributions is that the force arising from the pressure around the control surface is only 15% the scale of the inertial contribution. Hence, to leading order, the thrust produced by the nozzles is determined by the inertial term in the control volume equation (Eq. 3.3), so that:

$$T = A_n(\rho u_j^2 + p_n) \approx \rho u_{cv} \dot{V}_{cv}.$$
(4.1)

This data-driven approximation provides an interesting simplification to analyzing the mechanism behind thrust generation in a single jet. While it is understood that at the nozzle exit plane, an increased nozzle pressure provides a thrust benefit to using a pulsed jet [7, 8], this effect has been interpreted in terms of added mass effects and entrainment of ambient fluid [7, 8, 11, 12, 44]. These interpretations can be refined for the early times by observing that the pressure forces associated with ambient added mass are much smaller than the momentum change associated with entraining ambient fluid into the vortex bubble.

#### 4.2 The effect of two-jet interactions on thrust

Because the present analysis technique focuses on times where the control volumes remain nearly axisymmetric, the behavior of the dynamical quantities  $(u_{cv}, V_{cv}, a, b, z_{ell})$  are qualitatively similar to those presented for the single nozzle experiments. For this reason, most of the time series plots are omitted here. For the interested reader, additional time series plots for two-nozzle experiments are provided in Appendix C.

In order to understand the effect of nozzle proximity on thrust production, I compare the average values of the various measured and derived quantities at each value of  $\tilde{\Delta}$ . Because of the slight differences in the behavior of the two nozzles, quantities



Figure 4-3: Evolution of the vortex bubble geometry. As described in Chapter 3, the vortex bubble is fitted to an ellipse, and the ellipse's minor radius a and major radius b are tracked along with its center. The ellipse radii and their time derivatives, plotted here, are used to calculate the pressure along the leading edge of the vortex bubble.



**Figure 4-4:** Evolution of the vortex bubble position and  $\hat{\mathbf{z}}$  velocity. Like the CV velocity and volume, the vortex bubble position can be reasonably fit to a straight line to ignore the oscillations.

measured for nozzle 1 are provided relative to the average behavior of nozzle 1 individually (e.g.  $\overline{u}_{cv,1}(\widetilde{\Delta})/\overline{u}_{cv,\infty,1}$ ). The data for nozzle 2 is normalized in the same way. This normalization tends to collapse the behavior of both nozzles onto a single curve for any given quantity, providing more robust measurements of the trends associated with nozzle spacing. Un-normalized data are provided in Appendix C.

As in the single nozzle experiments, the CV position and volume were tracked for each nozzle in the two-nozzle experiments, and the time evolutions were fit to lines in order to determine the CV velocity and growth rate. These quantities, averaged over the entire experiment (from t = 0.08s-0.30s), and normalized for nozzle behavior, are plotted for each  $\tilde{\Delta}$  in Figs. 4-7 and 4-8.

From these figures it is apparent that while the CV velocity depends rather strongly on nozzle spacing, the volume growth rate is roughly constant across the range of  $\widetilde{\Delta}$ , with the exception of a couple of the points above  $\widetilde{\Delta} = 3.30$ . These points seem to be outliers, although the source of their deviation remains unknown. As with all quantities described in this section, the normalized values should asymptote to 1 as  $\widetilde{\Delta} \to \infty$ , since the single-nozzle experiments realize that limit. Therefore, since I have no reason to believe that there is a physical phenomenon causing the erratic



**Figure 4-5:** Pressure and inertial contributions to thrust for single-nozzle. Because the vortex bubble growth is not linear, the pressure force grows nonlinearly with time. When comparing the pressure forces to the inertial forces, observe that the scales on the vertical axes are different. The inertial forces are approximately ten times greater than the pressure forces at the times investigated.



Figure 4-6: Thrust evolution with time for single-nozzle experiments. The thrust scale is dominated by the (constant) inertial contribution from the CV analysis.



Figure 4-7: Normalized average CV velocity for each experiment, as a function of nozzle spacing  $\widetilde{\Delta}$ . The CV velocity exhibits a strong and highly local dependence on  $\widetilde{\Delta}$ , increasing approximately as  $1 - \widetilde{\Delta}^{-6}$ . The blue squares represent data from nozzle 1, red triangles represent nozzle 2, and the black line represents a fitted 1-parameter model as described in the text.



**Figure 4-8:** Normalized average CV volume growth rate  $\dot{V}_{cv}$  for each experiment, as a function of nozzle spacing  $\tilde{\Delta}$ .  $\dot{V}_{cv}$  remains nearly constant across all experiments, with the exception of some of the higher values of  $\tilde{\Delta}$ . The blue squares represent data from nozzle 1, red triangles represent nozzle 2. The shaded region represents the standard deviation of  $\dot{V}_{cv,\infty}$  measurements.

behavior around  $\widetilde{\Delta} = 3.3$ , I attribute the small (2%) deviation to some experimental factor. I will continue with the analysis of these experiments, aware of these unusual outliers.

The behavior of the CV velocity follows a trend that is reasonably described by:

$$F(\widetilde{\Delta}) = 1 - C\widetilde{\Delta}^{-6},\tag{4.2}$$

where F is the (normalized) quantity on the vertical axis and C is a dimensionless parameter that describes the strength of interaction between the nozzles. Fitting the normalized CV velocity to this 1-parameter model yields a coefficient of  $C_u = 1.94 \pm 0.06$ . The error bars around the fitted model reflect the standard deviation of the experimental data at  $\widetilde{\Delta} \to \infty$ . This empirical model is useful elsewhere in this section, and a justification for this form is presented in Chapter 5.

The different behavior of  $\overline{u}_{cv}$  and  $\dot{V}_{cv}$  reveals an interesting prediction for the thrust associated with vortex formation. Since  $\dot{V}_{cv}$  is nearly constant across all  $\widetilde{\Delta}$ , the approximation in Eq. 4.1 suggests that, with respect to its  $\widetilde{\Delta}$  dependence, thrust should scale as  $T \sim \overline{u}_{cv}$ . This prediction is validated by the data in Fig. 4-14.

The measurements for  $u_{cv}$  and  $\dot{V}_{cv}$  are combined to calculate the inertial contribution to force  $T_{II}$ , which is plotted in Fig. 4-11. Because  $u_{cv}$  contains the strongest dependence on  $\tilde{\Delta}$ , the calculation of  $T_{II}$  reflects this dependence most clearly. Fitting  $T_{II}$  to the form (Eq. 4.2) yields a coefficient  $C_{T_{II}} = 1.74 \pm 0.11$ .

Next, in order to calculate the pressure forces, the vortex bubble geometry was tracked, and the average principal radii growth rates were calculated ( $\dot{a}$  and  $\dot{b}$ ). The results of these calculations are plotted as a function of  $\tilde{\Delta}$  in Fig. 4-9. Similar to the CV velocity, both radii growth rates reveal a strong and highly local dependence on  $\tilde{\Delta}$ . An interesting result here is that the minor radius growth rate  $\dot{a}$  exhibits a much stronger dependence on  $\tilde{\Delta}$  than the major radius  $\dot{b}$ , and that the sign of



Figure 4-9: Average vortex bubble growth rates for the minor and major radii,  $\dot{a}$  and  $\dot{b}$ . As two nozzles are brought into closer proximity, the growth rate  $\dot{b}$  slows, and a concomitant increase in  $\dot{a}$  is observed.



Figure 4-10: Average vortex bubble velocity  $\overline{w}$ . As the nozzles are brought together, the velocity decreases significantly, reflecting the trend  $1 - C\widetilde{\Delta}^{-6}$ .



Figure 4-11: Normalized average inertial contribution to the thrust  $T_{II}$ , as a function of nozzle spacing  $\tilde{\Delta}$ . For close nozzle placement, the inertial force term drops by nearly 8% compared to the limiting value of a single nozzle. The outliers in  $\dot{V}_{cv}$  are reflected in the calculation of  $T_{II}$ .



Figure 4-12: Normalized average pressure contribution to the thrust  $T_{III}$ , as a function of nozzle spacing  $\tilde{\Delta}$ . As the nozzles are brought closer together, the average pressure force drops by nearly 30%. Despite this large fractional drop, because the pressure force is an order of magnitude smaller than the inertial force, the total thrust does not change as strongly.



Figure 4-13: Relative scale of the two contributors in the thrust calculation: pressure contribution to inertial component  $(T_{III}/T_{II})$ . As was seen for the single nozzle experiments, the scale of the pressure contribution is much smaller than that of the inertial contribution, indicating that entrainment and CV acceleration is a more important process than the ambient added mass reaction.

the dependence is opposite that of  $\dot{b}$  and  $\overline{u}_{cv}$ . As the nozzle spacing decreases, the minor radius grows up to 40% larger, and the major radius shrinks by nearly 15%. This result reflects the earlier observation that the wake maintains a nearly constant volume, as show in Fig. 4-8. Since a < b and  $V_{vb} = 4\pi a b^2/3$ , a small decrease in b requires a larger increase in a to maintain a constant volume.

In addition to the vortex bubble's growth rate, its position was measured and linearized to calculate the vortex bubble velocity w. The behavior of  $\overline{w}$  is qualitatively similar to that of  $u_{cv}$ , with a considerably stronger dependence on  $\widetilde{\Delta}$ . This difference in coupling strength arises because the CV average velocity includes regions that are stationary from frame to frame, while the vortex bubble is subset of the CV that is always moving. The vortex bubble velocity is plotted in Fig. 4-10. The coefficient fitted from the model is  $C_w = 6.37 \pm 0.20$ .

The time-evolution of the vortex bubble geometry and velocity are used to cal-

culate the pressure contribution to the total thrust  $T_{III}$ , which is plotted Fig. 4-12. Compared to the inertial contribution to the thrust  $T_{II}$ , the pressure contribution demonstrates a stronger dependence on  $\tilde{\Delta}$ , although the data is somewhat noisier. Fitting the model (Eq. 4.2) to the data provides a coefficient  $C_{T_{II}} = 1.74 \pm 0.11$  for the inertial contribution and  $C_{T_{III}} = 7.08 \pm 0.73$  for the pressure contribution.

Despite the stronger dependence of  $T_{III}$  on  $\widetilde{\Delta}$ , this behavior is not significantly reflected in the total thrust because the scale of the pressure force is significantly lower than that of the inertial force (similar to what was observed in the single nozzle experiments). Fig. 4-13 demonstrates this point by plotting the relative strength of the two thrust components  $T_{III}/T_{II}$  against the nozzle spacing  $\widetilde{\Delta}$ . This plot reveals that for all nozzle spacings,  $T_{III} \leq 0.15T_{II}$ .

Finally, the total thrust is calculated according to Eq. 3.3, and plotted as a function of  $\widetilde{\Delta}$  in Fig. 4-14. When the inertial and the pressure contributions are combined, the result is a strong, highly localized dependence on  $\widetilde{\Delta}$ . As the nozzles are brought into close proximity, the average thrust drops by nearly 10% of its value for a single nozzle. Such a large drop can have important implications for vehicle control when using multiple nearby thrusters based on a pulsed jet. These implications are discussed in the following chapter.

For now, it only remains to describe the form of the thrust dependence on  $\tilde{\Delta}$ . As was the case before, the thrust aligns well with the trend described by Eq. 4.2. As the nozzle spacing decreases below a critical value ( $\tilde{\Delta}_c \sim 2.5$ ), coupling between the two jets suddenly becomes more important. In this operating regime, jet coupling causes the thrust to fall as

$$\overline{T} \sim \overline{T}_{\infty} (1 - Co\tilde{\Delta}^{-6}), \tag{4.3}$$

where  $Co = 2.42 \pm 0.11$  is the dimensionless "coupling coefficient" that describes how much nozzle coupling affects thrust production relative to an individual nozzle.



**Figure 4-14:** Normalized average thrust over first 0.3s for each nozzle spacing tested. The normalized thrust should monotonically asymptote to 1 as  $\tilde{\Delta} \to \infty$ . The solid line represents a fit to the 1-parameter model given by Eq. 4.2, which is discussed further in Section 5. In this case,  $\overline{T}/\overline{T}_{\infty} = 1 - Co\tilde{\Delta}^{-6}$  for a value of the coupling coefficient,  $Co = 2.42 \pm 0.11$ . The shaded region represents the standard deviation of the average thrust measured for the single nozzle ( $\tilde{\Delta} \to \infty$ ). Blue squares represent the mean values from nozzle 1, and red triangles represent the mean values from nozzle 2. The results of individual experiments are shown as colored circles. In many cases, these points are covered by the markers for the average.

#### 4.3 Effects on efficiency

One important question that has not yet been addressed is effect of  $\tilde{\Delta}$  on the thruster's efficiency. For this purpose, recall that the conceptual thruster, the nozzle in my experiments, is driven by a pressure  $p_d$ , which displaces a volume at a rate  $\dot{Q}$  for a period dt. Then the power put into producing the thrust is  $\dot{E} = p_d \dot{Q}$ . As a measure of useful work production, I note that the goal of these thrusters is maneuverability - fast bursts of thrust for short periods of time. So a measure of useful work is the thrust produced T, multiplied by the speed at which it can be delivered  $u_j$ , so that the total efficiency for the thruster is be given by:

$$\eta = \frac{Tu_j}{p_d \dot{Q}}.$$

Plugging in typical experimental values:  $T \sim 0.05$ mN,  $u_j \sim 50$ mm/s,  $p_d \sim 114$ Pa, and  $\dot{Q} \sim 1.68$ mL/s, yields an abysmal efficiency of  $\eta \approx 0.013$ . However, this experiment was not designed to replicate a thruster in all regards. Many design parameters, including the thruster scale and operating regime, will affect the efficiency and should be accounted for in design.

The valuable result from these experiments is an understanding of how the nozzle spacing  $\widetilde{\Delta}$  affects the efficiency. Since the jet velocity  $u_j$ , flow rate  $\dot{Q}$  and driving pressure  $p_d$  did not vary with  $\widetilde{\Delta}$ , the efficiency of the thruster should scale as the thrust T does. Moreover, compared to the efficiency of a single jet  $\eta_{\infty}$ , the efficiency of two jets separated by a distance  $\Delta$  will be given by:

$$\frac{\eta(\widetilde{\Delta})}{\eta_{\infty}} = \frac{T(\widetilde{\Delta})}{T_{\infty}} = 1 - Co\widetilde{\Delta}^{-6}.$$
(4.4)

Based on this result, Fig. 4-14 not only indicates how thrust varies with nozzle spacing, but also how the efficiency should vary with nozzle spacing as well.

## Chapter 5

## Discussion of two-nozzle interactions in pulsed jet formation

Based on the results presented in Chapter 4, when two simultaneously pulsed jets are brought into close proximity, their wake dynamics destructively interfere with each other, causing a drop in the total thrust generated as well as in the efficiency of the thrust production.

I use this chapter to describe a possible physical mechanism for thrust and efficiency reduction with  $\tilde{\Delta}$ . This model provides insights for both vortex formation dynamics and for thruster design. In the first section, I discuss how my data reveals physical mechanisms relevant to thrust production in pulsed jets. Then, in the remaining two sections, I develop two scaling arguments based on physical principles that qualitatively account for the behavior of T shown in Fig. 4-14.

Throughout the discussion in this chapter, I assume that ambient pressure is  $p_0 = 0$ , or equivalently, I treat all pressures as gauge pressures.

# 5.1 Sources of nozzle over-pressure during vortex formation

Data obtained by a control volume analysis can often be tricky to interpret. While control volumes are a useful analysis tool that can instruct indirect measurement techniques, the results of the analysis are only correlations - they do not reveal any causal dependence between the quantities measured. Consider, for example, the thrust equation, Eq. 3.3, which allows the total force to be measured by analyzing fluorescence data (reproduced here for convenience):

$$T = A_n \rho u_j^2 + A_n p_n \approx \underbrace{\rho \dot{\overline{u}}_{cv} V_{cv}}_{\mathrm{I}} + \underbrace{\rho \overline{u}_{cv} \dot{V}_{cv}}_{\mathrm{II}} + \underbrace{\int_{S_{vb}} p_{vb} \, \hat{\mathbf{n}} \cdot \hat{\mathbf{z}} \, dA}_{\mathrm{III}}.$$

On its own, this equation does not explain any physical mechanism behind thrust production in a pulsed jet; it can only indicate how these groups of terms are related. In the most fortunate circumstances, the analysis can reveal that one or two of the measured terms dominate the behavior, and that the rest can be ignored. Such was the case in my experiments, whereby the first and third terms on the right side of the equation can be neglected compared to the second term, when considering only leading order behavior. When further coupled with additional data and a model, the control volume approach can become very powerful. For the data presented herein, some illuminating physics can be drawn from these empirical observations coupled with models that describe the physical relationship between different terms when each term can be associated with a different physical phenomenon, the relative magnitudes of the terms can illuminate which phenomena are most closely linked in a complex system such as vortex formation.

To see this directly consider the physical interpretations of each term in the thrust equation:

- T The total thrust experienced by the thruster, which is equivalently the rate of momentum flux into the wake.
- $A_n \rho u_j^2$  This term represents the component of thrust associated with generating fluid motion at a velocity  $u_j$  within the thruster.
- $A_n p_n$  This term is the force given by nozzle over-pressure, which arises from near-field dynamics as the vortex is forming. It can be derived explicitly in terms of the streamline curvature at the nozzle exit [10], and has been interpreted in terms of the added mass of the forming vortex wake [7, 8].
- $\rho \bar{u}_{cv} V_{cv}$  This term represents the force required to instantaneously accelerate the control volume's center of mass.
- $\rho \overline{u}_{cv} V_{cv}$  This term represents force required to bring external fluid into the CV, accelerating it to speed  $\overline{u}_{cv}$ . The term  $\rho \dot{V}_{cv}$  includes both the volume growth because of mass injection from the jet  $(\rho u_j A_n)$ , as well as volume growth from ambient entrainment via vortex roll-up (shear driven).
- $\int_{S_{vb}} p_{vb} \ dA_z \qquad \text{This term comes from the pressure resisting control volume growth and} \\ \text{motion (here, in the } \hat{\mathbf{z}} \text{ direction}). \text{ This pressure arises as an added mass} \\ \text{reaction associated with accelerating all of the fluid originally outside} \\ \text{of the nozzle. In a common interpretation, this term represents the jet} \\ \text{`pushing off' of the ambient fluid.} \end{cases}$

Starting at the beginning of the thrust equation, a control volume around the thruster itself (Fig. 3-4) indicates that there are two contributors to thrust production: the jet velocity (inertia),  $u_j$ , and the nozzle pressure  $p_n$ . Currently, pulsed jets are understood to produce more thrust than equivalent continuous jets because they gain the benefit of the additional nozzle pressure  $p_n$ . Mathematically, this nozzle pressure can be related to the curvature of streamlines near the nozzle [10], where the vortex

is rolling up during pulsed jet formation. Revisiting the data in Fig 4-14 in this light, it becomes clear that any effects of jet interactions on thrust reflect the effects that the interactions have on the nozzle over-pressure  $p_n$ . To see this, recall that across all of the experiments, the jet velocity  $u_j$  did not vary, so any variation observed in Tmust arise solely from variations in  $p_n$ . In this case, with respect to the scaling with  $\widetilde{\Delta}$ , the first half of Eq. 3.3 reduces to

$$T \sim A_n p_n$$

This in mind, it remains to understand what physical interactions arise between neighboring nozzles to affect  $p_n$ . To do this, the second control volume around the wake is useful, which contributes the final three terms to Eq 3.3. Based on the interpretations provided above, terms I and II represent the inertial portion of the CV momentum, and term III represents the pressure portion, associated with external added mass.

As has been commonly explained based on control volume analyses [7, 8], one source of the thrust benefit associated with pulsed jets has been explained as the jet 'pushing off' of the stationary fluid behind the jet (term III). Another source of the thrust benefit has been described in relation to the external fluid entrained into the vortex bubble (terms I and II) [7]. While these two interpretations are not mutually exclusive, they do represent different physical phenomena that can contribute to thrust production (in fact these two phenomena must arise together when conservation of mass and vorticity are accounted for in the unsteady flow). The added mass term is dominated by normal forces generated since the pulsed jet must accelerate previously stationary flow immediately around the nozzle. Krueger uses a potential flow model to describe how this term must be significant at very early times during formation [7]. The inertial terms, on the other hand, is produced by normal and shear forces as the jet grows into the ambient fluid, and also entrains (shear) ambient fluid into the CV. Given the data presented in Fig. 4-2, it is clear that during the course of these experiments, the control volume doesn't accelerate significantly, so that  $\dot{\bar{u}}_{cv} \approx 0$ as observed in Chapter 4. This observation indicates that the only inertial contribution comes from term II, which represents CV growth. From this simplification, thrust production must be associated with these two phenomena:

$$T = \rho u_{cv} \dot{V}_{cv} + \int_{S_{vb}} p_{vb} dA_z.$$

The two remaining terms associated with the wake inherently represent the two different mechanisms for thrust production: added mass associated with the jet 'pushing off' of the ambient fluid, and shear-driven entrainment of ambient fluid into the CV.

While both interpretations have been used to describe the benefits of pulsed jet thrust production, the data presented in Fig. 4-13 clearly demonstrates that the more important effect comes from the entrainment term given that  $T_{III}/T_{II} < 0.15$  for all experiments. Moreover, by observing that the total volume growth rate  $\dot{V}_{cv}$  does not change with  $\tilde{\Delta}$ , and the previous observation that  $u_j$  doesn't change with  $\tilde{\Delta}$ , the thrust scaling becomes

$$T(\widetilde{\Delta}) \sim A_n p_n(\widetilde{\Delta}) \sim \rho \dot{V}_{cv} u_{cv}(\widetilde{\Delta}).$$
(5.1)

Now that this scaling relationship is reduced to only one term on each side, the complex physical relationships in the problem of pulsed jet thrust production can be decoupled. The simplest explanation of Eq. 5.1 indicates that additional nozzle pressure does not change the amount of volume entrained into the CV as the wake develops (this is likely set by the circulation, which is set by  $u_j$ ), but does determine how fast the CV is moving.

## 5.2 Geometric argument for thrust reduction based on the slug model

Despite its implications for understanding wake dynamics in the pulsed jet, the result in Eq. 5.1 does not intuitively reveal why  $p_n$  decreases as  $\tilde{\Delta}$  is decreased.<sup>1</sup> To develop a better model for this, I resort to a scaling argument based on an observation made by Krueger [19]. He demonstrated that the nozzle over-pressure  $p_n$  is dependent on the curvature of the streamlines at the nozzle exit plane according to:

$$p_n \sim p(r=0, z=0) = \rho \int_0^{D/2} u_z \frac{\partial u_r}{\partial z} \Big|_{z=0} dr,$$
 (5.2)

where  $u_z$  is the axial velocity field and  $u_r$  is the radial velocity field, evaluated at the nozzle exit plane z = 0. Given this relationship, it makes sense that any physical obstacle or interaction that changes the streamlines, specifically that alters the axial gradient in radial velocity, will affect the nozzle over-pressure. If two ideal pulsed jets are placed next to each other, then they represent mirror copies of each other so that the symmetry condition  $u_{\perp} = 0$  must be satisfied on the plane dividing the two nozzles. In this case, then, the radial growth of the vortex ring is limited, and by volume conservation, it must grow axially instead. Growing axially instead of radially decreases  $\partial u_r/\partial z$ , thereby decreasing the nozzle pressure.

To verify how well this geometric model is supported by the data, consider how the terms in Eq. 5.2 scale with the experimentally measured quantities. From the problem definition, it is natural to expect  $u_z \sim u_j$ , and  $r \sim D$ . But for the  $u_r$  and z, the scalings are less obvious. Because the only measured motion in the  $\hat{\mathbf{r}}$  direction

<sup>&</sup>lt;sup>1</sup>It is tempting to claim that two vortices next to each other will induce a counter flow that slows the wake of the neighboring flow by a factor of  $u_{ind}$ , thereby decreasing nozzle pressure by nature of Eq. 5.1. However, causality appears backwards in this argument - I would expect a reduced wake velocity should not cause the nozzle pressure to increase. To the contrary, a slower wake could arise from higher ambient and nozzle pressures at fixed jet velocity! Furthermore, the functional form of  $u_{ind}$  does not match well with the observed data. This line of thinking, however, is useful if instead the induced *pressure* is considered. See Section 5.3 and Appendix B for more on this argument.

is associated with the lateral radial growth of the vortex bubble, it is reasonable to expect that  $u_r \sim \dot{b}$ . The derivative  $\partial u_r/\partial z$  represents how suddenly (along z) the radial velocity changes. Given the geometry of the forming vortex ring, the radial velocity must change from  $u_r = 0$  to its maximum value between the nozzle exit plane and the leading edge of the vortex bubble. This distance is given exactly by  $a + z_{ell}$ , making this a reasonable choice for the z scale. So the final scaling relationships are

 $u_z \sim u_j$   $r \sim R = D/2$   $u_r \sim \dot{b}$  and  $z \sim (a + z_{ell})$ .

These scalings lead to the expectation that

$$p_n \sim \rho u_j R \frac{\dot{b}}{a + z_{ell}}.$$
(5.3)

Since  $u_j$  is constant with  $\widetilde{\Delta}$ , and  $\dot{b}$  drops with  $\widetilde{\Delta}$  while *a* increases with  $\widetilde{\Delta}$ , it seems that the data support this scaling argument, and hence the explanation that nozzle interactions lower nozzle pressure by geometrically enforcing straighter streamlines. To see the agreement more directly, Fig. 5-1 shows  $p_n$  plotted against  $\widetilde{\Delta}$  as it is scaled in Eq. 5.3. The model fit to this scaling demonstrates that  $p_n \sim p_{n,\infty}(1 - C_p \widetilde{\Delta}^{-6})$  for a pressure coupling coefficient  $C_p = 2.77 \pm 0.03$ .

Inserting this nozzle pressure scaling into the thrust equation yields a prediction for the value of the thrust coupling coefficient in terms of the (independently measured) pressure coupling coefficient:

$$T(\widetilde{\Delta}) = T_{\infty}(1 - Co\widetilde{\Delta}^{-6}) = A_n(\rho u_j^2 + p_{n,\infty} - p_{n,\infty}C_p\widetilde{\Delta}^{-6})$$
$$= A_n(\rho u_j^2 + p_{n,\infty}) \left(1 - \frac{p_{n,\infty}}{\rho u_j^2 + p_{n,\infty}}C_p\widetilde{\Delta}^{-6}\right)$$
$$\approx T_{\infty} \left(1 - \frac{1}{2}C_p\widetilde{\Delta}^{-6}\right).$$



Figure 5-1: (left) Estimated nozzle pressure  $p_n$  as a function of nozzle spacing  $\Delta$ , calculated based on the geometric scaling described in Sec. 5.2. As opposed to the calculation of total thrust, which involved volume measurements and potential flow estimates, the values in this plot were based *solely* on the geometry and motion of the vortex wake. The nozzle pressure changes with  $\tilde{\Delta}$ as  $p_n/p_{n,\infty} = 1 - C_p \tilde{\Delta}^{-6}$  for a pressure coupling coefficient of  $C_p = 2.77 \pm 0.03$ . (right) The scaling estimates were calculated for each moment throughout the experiment, and these time-series are shown for select values of  $\tilde{\Delta}$ . To arrive at the values plotted in the left plot, the average value of each time series was calculated.

Here, the approximation in the final step is made based on Krueger's data that for low stroke ratios, the nozzle pressure contributes approximately half of the total thrust produced [7, 8], so that  $p_{n,\infty}/(\rho u_j^2 + p_{n,\infty}) \approx 1/2$ . This analysis predicts a value of the thrust coupling coefficient

(geometric prediction) 
$$Co = C_p/2 = 1.39.$$

Recall that the experimental measurements of Co (which were calculated through different means) indicate that Co = 2.42 (see Fig. 4-14). It seems that the scaling model under-predicts the effect of spacing on nozzle pressure. However, as it is only off by an order 1 constant, such alignment of the scaling prediction compared to the thrust measurement suggests that this scaling model reasonably explains the physical mechanisms at that cause thrust reduction.

Hence, the thrust reduction observed with decreasing  $\widetilde{\Delta}$  can be explained by geo-

metric constraints on the streamlines during formation. Because the two neighboring nozzles create a geometric constraint on each others' growth (perhaps explained by interacting pressure fields), they force the streamlines to straighten, reducing nozzle pressure. Tying this back into the other observations, because the nozzle pressure is reduced, the wake is driven away from the nozzle more slowly than it would have in the absence of the neighboring nozzle.

The physics of this model are further validated when compared to the results of Krueger, Dabiri, and Gharib in their experiments of vortex formation in ambient coflow and counter-flow [13, 14]. In the case of ambient co-flow (flow parallel to jet), my model would predict that additional streamwise motion should increase a, decreasing b as a result of continuity. Based on these assumptions, the scaling model predicts that the nozzle pressure should decrease, thereby lowering the production of thrust and circulation as is reported by Krueger et al. [13]. In the case of ambient counterflow (flow antiparallel to the jet), the predictions would reverse, suggesting that adecreases and b should increase by continuity. In that case, the geometric model would predict a higher nozzle pressure, thrust and circulation production, which is reflected in the data from Dabiri and Gharib [14].

## 5.3 Scaling argument for the functional form of thrust reduction: $T \sim T_{\infty}(1 - Co\widetilde{\Delta}^{-6})$

While the geometric argument provides an interesting way to predict that thrust (and efficiency) should decrease as nozzle spacing is decreased, the functional form ( $\sim \tilde{\Delta}^{-6}$ ) is difficult to derive that way. However, another scaling approach can be used to more clearly derive the functional dependence of T on  $\tilde{\Delta}$ .

Because  $T(\widetilde{\Delta}) = A_n(\rho u_j^2 + p_n(\widetilde{\Delta}))$ , any changes in T should arise because of an additional pressure scale  $\tilde{p}$  that is introduced by a second nozzle. Additionally, I

expect that this pressure scale should shrink as  $\widetilde{\Delta}$  grows so that as  $\widetilde{\Delta} \to \infty$ , the additional pressure scale  $\tilde{p} \to 0$ , and  $T \to T_{\infty}$ .

These requirements in mind, the pressure scale that I suggest arises comes from the velocity induction associated with the forming vortex ring. It can be shown that a single vortex ring with circulation  $\Gamma \sim u_j^2 t$  induces a velocity that scales as:

$$u_{ind} \sim u_j \frac{u_j t}{D} \left(\frac{D}{r}\right)^3.$$

See Appendix B and Lamb [49] for more details. Here r is the distance from the vortex ring in the same (z = 0) plane as the ring. So at a distance  $r = \Delta$  from the nozzle center, the associated induced velocity will be

$$u_{ind} \sim C\widetilde{\Delta}^{-3}$$

where the constant C incorporates all of the terms from the equation for induced velocity that don't scale with  $\widetilde{\Delta}$  (including  $u_j$  based on the data in Chapter 4).

The pressure scale associated with this velocity field is the stagnation pressure

$$p_{ind} = \rho u_{ind}^2 \sim \mathcal{C}^2 \widetilde{\Delta}^{-6} \tag{5.4}$$

Based on the argument in Section 5.2, I expect the effect of nozzle interactions is to *lower* the nozzle pressure by the induced pressure scale, so that when two pulsed jets are brought together, the nozzle pressure for a single jet  $p_{n,\infty}$  is modified as

$$p_{n,\infty} \to p_{n,\infty} - p_{ind}.$$

Then the thrust can be expected to scale as

$$T(\widetilde{\Delta}) \sim A_n(\rho u_j^2 + p_{n,\infty} - p_{ind})$$
  
 
$$\sim A_n(\rho u_j^2 + p_{n,\infty} - \mathcal{C}^2 \widetilde{\Delta}^{-6})$$
  
 
$$\sim A_n(\rho u_j^2 + p_{n,\infty})(1 - Co \widetilde{\Delta}^{-6}),$$

where the coupling coefficient Co is defined here in terms of  $\mathcal{C}$ ,  $p_n$ ,  $\rho$ , and  $u_j$ .

Finally, recognizing the thrust produced by a single jet  $T_{\infty} = A_n(\rho u_j^2 + p_{n,\infty})$ , the thrust can be expected to scale as

$$T \sim T_{\infty} (1 - Co\tilde{\Delta}^{-6}) \tag{5.5}$$

as observed empirically in Chapter 4.

#### 5.4 Final remarks on the scaling of T with $\Delta$

Given the data and scaling arguments presented above, the problem remains to determine what sets the coupling coefficient Co. In terms of the arguments presented herein, how does the geometric mechanism described in Section 5.2 tie into the model presented here to produce the coupling number Co. Are there system configurations that can allow Co < 0? Such behavior would be welcomed because it would provide a means to increase the pulsed jet's thrust and efficiency.

One way to possibly accomplish this can be garnered from the argument that thrust reduction arises because streamlines are confine to straighter paths. If instead of being ejected at the same time, jets were pulsed so that the second jet is ejected as the stopping vortex forms in the first jet, the close proximity of the negative vorticity from the first jet would *help* to curve the streamlines exiting the second nozzle, thereby augmenting the nozzle over-pressure. In this way, well-timed pulses could exploit the stopping vorticity in each others' wakes to roll-up more efficiently and produce more thrust. This behavior is not unlike how jellyfish exploit stopping vortices to move more efficiently [35].

When considering how this behavior scales to larger systems with more nozzles, I believe that the analysis provided in this section is generalizable to more nozzles. The geometric and pressure scalings should still apply, since more nozzles will further straighten the flow, and will provide additional pressure scales that add linearly to the nozzle over-pressure. The most significant difference will be observed because of the 3D nature of the wake development. While the pressure scales can be added linearly, and treated pairwise, the streamlines will develop based on a more complex three-dimensional flow field around the nozzle, and this behavior cannot be predicted by the current analysis.

On of the largest limitations of the present analysis is that it assumes quasi-2D wake development. In reality, the wake dynamics very rapidly become more complex three-dimensional structures as two nozzles are brought close together and axisymmetry is broken. It is likely that arguments presented here will continue to offer leading-order insights into the problem, but a more complete understanding of the wake dynamics – including the behavior of vorticity and transport properties of the flow – will require a more thorough 3D investigation.

#### 5.5 Implications for continuously pulsed jets

Although my experiments did not directly investigate the effects of nozzle proximity on continuously pulsed jets, the results could be extended to the continuous case using the analytical results of Ruiz et al.. Using an assumption of steady vehicle motion and high pulse frequency, Ruiz et al. demonstrated that the hydrodynamic
efficiency of the pulsed jet vehicle can be described by [25]

$$\eta_{RWD} = (1 + \alpha_{xx}^{VB}) \frac{u_{\infty}}{w}, \qquad (5.6)$$

where  $\alpha_{xx}^{VB}$  is the Bessel added mass associated with the vortex bubble,  $u_{\infty}$  is the vehicle speed, and w is the vortex bubble velocity. The added mass term for any ellipsoidal vortex ring can be calculated from vortex bubble geometry using the potential flow result described in Appendix A. Since the vortex ring added mass decreases between experiments, and the vortex bubble velocity decreases by as much as 25% as the nozzles are brought closer together, this analysis suggests the possibility that two-nozzle interactions could provide an efficiency *boost* during continuous motion. This result requires further investigation, but may indicate key differences between optimal propulsive techniques for steady motion required by long-range travel and the impulsive motion required by position control systems.

#### 5.6 Salps: swarming jets of the sea

This project was originally inspired by the behavior of salps, so it seems appropriate to conclude with a few comments on their behavior. Salps are a marine invertebrate shaped like a hollow tube (see Fig. 5-2a). Salps can range in sizes, from roughly 0.5-10cm in length and about 0.1-3cm in diameter [2, 3]. They propel themselves through the water by opening their atrial siphon and compressing their bodies using muscles that cause a radial contraction. This motion forces a short pulsed jet out of their rear opening, by which they generate forward thrust. To recover the water lost during ejection, salps then open their front lips and draw in water from the oncoming stream [2]. This process typically repeats at a frequency around f = 1Hz for forward swimming [2, 3].

Compared to other underwater creatures that move around using pulsed jets (such



Figure 5-2: Images of salps (*Thalia Democratia*) in the ocean. (a) Solitary salp. (b) Chain of three salps. Images reproduced from Cifonauta database [50, 51].

as jellyfish and squid), salps differ in their naturally arising community behavior. At certain points during their lifecycle, salps can be found as chains of individual animals that work together in order to propel the entire chain forward [2–4] (see Fig. 5-2b for an example of a short chain). These chains can range in size from a few individuals to hundreds of individual salps. For typical migratory swimming, salps pulse their jets in sequence down the chain, so that no two members eject a jet at the same time. However, when startled, the chain's escape response involves all members of the chain pulsing together [2]. While it is presumably less efficient for the chain to pulse together, because the periodic acceleration and deceleration loses more energy to drag, the escape response provides higher instantaneous acceleration and maneuverability to the chain.

By combining existing research on salps with research on continuously pulsed jets, one of the conclusions that can be drawn is that the chains will move most efficiently when the entire chain is moving steadily. In order to provide steady motion, the average pulse frequency of the chain should be as high as possible, suggesting that individual members of the chain should pulse in sequence. This way, even though any individual member is only expending the energy to pulse at a frequency f, the entire chain moves based on a group pulse frequency of Nf where N is the number of chain members. In this fashion, the chain can maneuver with the efficiency and lower drag associated with steady jetting, while simultaneously receiving the thrust benefits associated with pulsed jets.

Until now, this argument has provided the sole explanation for why salp chains pulse synchronously during migratory swimming. However, the results from my experiments reveal an additional, albeit secondary, benefit. For salp chains where individual members are closely aligned, my data suggests that individual members will produce thrust most efficiently when they *do not* pulse together. When considering the geometric model of thrust and efficiency loss, this observation has an interesting corollary for salp chains. Based on the geometric model that predicts how streamline curvature determines thrust loss, another prediction for salps is that if individual members are offset in the chain, such as the chain in Fig. 5-2, then consecutive chain members should not eject a jet until the wake from the neighbor has passed far enough away that it will not interfere with their own thrust production.

With more research, these observation can be tested and extended to the eventual design and control of marine robotic swarms. For small marine robots, efficient maneuverability remain two of the most challenging problems. By leveraging these results for the effects of nozzle spacing, in conjunction with observations of salp chains, marine robots can be designed and controlled for the most efficient maneuverability.

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#### Chapter 6

#### Summary and Conclusions

This thesis contains a first look at the effect of nozzle spacing on pulsed-jet propulsion. While there have been many studies on the dynamics and thrust production associated with pulsed jets, I am not aware of any other investigations of side-by-side interactions of vortex rings during formation. As such, this thesis has provided useful data explaining the relationship between different factors associated with early-time momentum transfer in a pulsed jet. These results can be used to help guide theories of vortex formation, and can inform the design and control of underwater vehicles.

In this thesis, I developed a visual force estimation technique to evaluate how thrust and efficiency of pulsed jets are affected by the placement of jets in a system. I validated that the technique calculated forces of the expected scale, and used the results to determine the scaling relationships between thrust, efficiency, and jet spacing. The results of this analysis immediately suggest certain design rules for the implementation of multi-jet marine thrusters and marine vehicle swarms. Most clearly, the results demonstrate that when parallel jets are simultaneously pulsed two interfering wakes are produced, which reduce the thrust production in each jet by as much as 10%. However, this effect appears to be highly localized so that once the nozzle spacing exceeds about two and a half nozzle diameters ( $\tilde{\Delta} > 2.5$ ), the effects on thrust and efficiency are negligible. Beyond the design recommendations, I used the results of these experiments to develop a description of the physical interactions governing simultaneous vortex ring formation. I used a scaling argument to demonstrate that bringing nozzles together causes thrust production to drop. This argument suggests that geometric constraints on wake development straighten streamlines at the nozzle, lowering nozzle overpressure.

Then, to determine the functional relationship between T and  $\tilde{\Delta}$ , I resorted to a pressure scale argument. I argued that in the case of simultaneous jet formation, the nozzle pressure should be modified by the pressure scale associated with the vortex ring that is forming at the neighboring nozzle. Combining these two arguments, I predicted a value for the coupling number Co, which governs how strongly the nozzle spacing reduces the thrust. Using separate methods to calculate thrust, I validated that these predictions support the experimental results, indicating that they reasonably describe the leading-order effects of vortex interactions during pulsed jet formation.

The results of this research not only point to a physical mechanism for thrust reduction in pulsed jets, but also suggest possible mechanisms for thrust augmentation in similar system configurations. Further research is necessary to determine whether different geometric design, or careful control of the jet timing can lead to thrust and efficiency *augmentation* for closely-spaced parallel pulsed jets. These results will have significant implications for understanding the behavior of marine animals such as salps, and for the design and control of future underwater robotic vehicles. Appendices

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#### Appendix A

## Pressure distribution around a translating and growing ellipsoid

This approach was inspired by the work of Munk on potential flow around ellipsoids for airship design [52].

As given by Lamb, the velocity potential for a translating ellipsoid with principal radii a in the  $\hat{\mathbf{z}}$  direction, b in the  $\hat{\mathbf{y}}$  direction, and c in the  $\hat{\mathbf{x}}$  direction, moving at velocity w in the  $+\hat{\mathbf{z}}$ -direction is most efficiently described in ellipsoidal coordinates [49]:

$$\phi(x,y,z,t) = -z \cdot \underbrace{\frac{w}{2-\alpha_0}}_{C(t)} \cdot \underbrace{abc \int_{\lambda}^{\infty} \frac{d\lambda'}{(a^2+\lambda')^{3/2} (b^2+\lambda')^{1/2} (c^2+\lambda')^{1/2}}}_{\alpha(\lambda,a,b,c;t)}.$$
 (A.1)

In Eq. A.1,  $\lambda$  is the ellipsoidal coordinate that grows perpendicular to the ellipsoid with principal radii (c, b, a) along (x, y, z) respectively. As shorthand for the different terms in Eq. A.1 I introduce the definitions:

$$C(t) = \frac{w}{2 - \alpha_0},\tag{A.2}$$

$$\alpha_0 = \alpha(0, a, b, c; t), \text{ and}$$
(A.3)

$$\alpha(\lambda, a, b, c; t) = abc \int_{\lambda}^{\infty} \frac{d\lambda'}{(a^2 + \lambda')^{3/2} (b^2 + \lambda')^{1/2} (c^2 + \lambda')^{1/2}}$$
(A.4)

It should be noted that x, y, z and  $\lambda$  are defined with respect to the moving ellipsoid center. It therefore makes sense to consider the equivalent problem of the ellipsoid in an unsteady free-stream flow with changing velocity  $-w(t) \hat{\mathbf{z}}$ .

A point in ellipsoidal coordinates is defined by the variables  $(\lambda, \mu, \nu)$  such that surfaces of constant  $\lambda$  are ellipsoids offset from the primary ellipsoid with principal radii (c, b, a) in the (x, y, z) directions respectively. To convert a point (x, y, z) from cartesian coordinates into ellipsoidal coordinates, the variables  $\lambda, \mu, \nu$  can be found as the solutions of the third-order polynomial in k defined by:

$$\frac{z^2}{a^2+k} + \frac{y^2}{b^2+k} + \frac{x^2}{c^2+k} - 1 = 0.$$

Canonically, the solutions for k are sorted so that  $\lambda > \mu > \nu$ . With this sorting,  $\lambda$  represents the spatial variable that increases perpendicular to the ellipsoid surface; surfaces of constant  $\lambda$  take the form of ellipsoids with scaled principle radii. The surface  $\lambda = 0$  lays coincident with the surface of the ellipsoid (c, b, a).

For the case of an oblate ellipsoid of revolution (about the  $\hat{\mathbf{z}}$  axis) with a < b, b = c and  $x^2 + y^2 = r^2$ , this equation reduces to:

$$\frac{z^2}{a^2+k} + \frac{r^2}{b^2+k} - 1 = 0.$$
 (A.5)

Since I am only interested in the pressure on the surface ( $\lambda = 0$ ) of such an oblate, axisymmetric ellipsoid translating along  $\hat{\mathbf{z}}$ , I can also use these assumptions

to simplify Eq. A.1:

$$\phi(x, y, z, t) = -z \cdot C(w, a, b; t) \cdot \alpha(\lambda = 0, a, b; z, r, t).$$
(A.6)

Furthermore, this simplification admits the following closed-form solution to the integral:

$$\begin{aligned} \alpha(\lambda, a, b; t) &= ab^2 \int_{\lambda}^{\infty} \frac{d\lambda'}{(a^2 + \lambda')^{3/2} (b^2 + \lambda)} \\ &= ab^2 \left[ -\frac{\pi}{(b^2 - a^2)^{3/2}} + \frac{2}{(b^2 - a^2)(a^2 + \lambda)^{1/2}} + \frac{2\tan^{-1}\left(\sqrt{\frac{a^2 + \lambda}{b^2 - a^2}}\right)}{(b^2 - a^2)^{3/2}} \right]. \end{aligned}$$
(A.7)

The integral  $\alpha$  relates to the Bessel added mass  $k_z$ , for the ellipsoid moving in the *z*-direction, according to [49]:

$$k_z = \frac{\alpha_0}{2 - \alpha_0} \tag{A.9}$$

Using this relationship, it can be shown that the closed form solution Eq. A.8 approaches the expected limits as  $a \to 0$  and  $a \to b$ , corresponding to a flat plate and a sphere respectively.

From these simplifications, it is possible to explicitly calculate the pressure on the surface of the vortex bubble using only the measured motion and growth of the ellipsoid bounding the vortex bubble. To calculate the spatial derivative required by the I equation (Eq. 3.4) I observe that the only spatially dependent terms in Eq. A.6 are C and  $\alpha$  so that:

$$\nabla\phi(t)\Big|_{S_{vb}} = -\hat{\mathbf{z}} \cdot C(w, a, b; t) \cdot \alpha_0(a, b, t) - z \cdot C(w, a, b; t) \cdot \frac{\partial\alpha}{\partial\lambda}\Big|_{\lambda=0} \cdot \left(\hat{\mathbf{z}}\frac{\partial\lambda}{\partial z}\Big|_{\lambda=0} + \hat{\mathbf{r}}\frac{\partial\lambda}{\partial r}\Big|_{\lambda=0}\right).$$

The derivative  $\partial \alpha / \partial \lambda$  can be evaluated by applying the fundamental theorem of calculus to Eq. A.7. The spatial derivatives of  $\lambda$  can then be calculated by implicitly differentiating Eq. A.5 with  $k \equiv \lambda$ .

To calculate the time derivative of the velocity potential on the surface of the ellipsoid, the chain rule can be applied to  $\phi$ :

$$\frac{\partial \phi}{\partial t}\Big|_{S_{vb}} = -z \cdot \alpha(0, a, b; t) \cdot \left(\frac{\partial C}{\partial w} \dot{u}_{vb} + \frac{\partial C}{\partial a} \dot{a} + \frac{\partial C}{\partial b} \dot{b}\right) - z \cdot C(w, a, b; t) \cdot \left(\frac{\partial \alpha}{\partial a} \dot{a} + \frac{\partial \alpha}{\partial b} \dot{b}\right)\Big|_{\lambda=0}.$$

Here, derivatives with respect to w, a, and b can be evaluated using Eqs. A.2, A.3, and A.8.

### Appendix B

## A pressure-based model to describe two-jet interaction

To model the interaction of two jets, I build off of the literature on vortex ring interaction and consider the effects of mutual induction between the two developing rings. In order to describe leading-order effects of these interactions, I do not consider the geometric deformations associated with the induction (for the times I experimentally analyze, these deformations are minimal anyway). Instead, I focus on the pressure fields that are set up by the forming vortex rings, which would establish the typical vortex-ring flow fields. Using this pressure scale as a modifier for the nozzle pressure explains why T should scale as  $\tilde{\Delta}^{-6}$ , as shown in Chapter 4.

Furthermore, as a leading-order approximation for the circulation produced by a pulsed jet, I refer to the 'slug-model' of pulsed jet ejection [7], so that the circulation produced is described by

$$\Gamma(t) = u_j^2 t.$$

See Fig. B-1 for a diagram of the system geometry. In cylindrical coordinates  $(r, \theta, z)$ , a 3-dimensional vortex ring of radius a, centered at the origin, with circulation



Figure B-1: (a) Induction Geometry. (b) While the induced velocity is described by a complicated equation, for the values of r that I am interested in (r/2a > 1.5), the induced velocity is well approximated by the scaling  $(r/2a)^{-3}$ .

 $\Gamma$  induces a flow field defined by the stream function [49]

$$\psi(r,z) = -\frac{\Gamma}{2\pi}(r_1 + r_2) \left[K(\lambda) - E(\lambda)\right]$$

where

$$\lambda = \frac{r_2 - r_1}{r_2 + r_1}$$
$$r_1^2 = z^2 + (r - a)^2$$
$$r_2^2 = z^2 + (r + a)^2$$

As shown in Fig. B-1,  $r_1$  is the closest distance of the vortex ring to the point (r, z), and  $r_2$  is the farthest distance. E and K are complete elliptic integrals of the first and second kind, respectively.

Since the two vortex rings are being formed simultaneously, I consider that their mutual effects are dominant on the plane z = 0. In this case, the induced velocity is antiparallel to  $\hat{\mathbf{z}}$ ,  $r_1 = r - a$ ,  $r_2 = r + a$ , and  $\lambda = a/r$ . Rewriting the ring radius

in terms of the nozzle diameter  $(a \rightarrow D/2)$  and differentiating the stream function yields a result for the velocity induced by a developing vortex ring.

$$\mathbf{u}_{ind}(r, z = 0) = \frac{1}{r} \frac{\partial \psi}{\partial r} \hat{\mathbf{z}}$$
$$= -\frac{\Gamma(t)}{\pi} \frac{1}{r} \left[ K\left(\frac{D}{2r}\right) - \frac{2 - \frac{D}{2r}}{2(1 - \frac{D}{2r})} E\left(\frac{D}{2r}\right) \right] (-\hat{\mathbf{z}}), \qquad (B.1)$$

Note that the time dependence is implicitly contained in the circulation,  $\Gamma$  since the circulation is increasing as the nozzle continues to provide fluid.

For large values of r, when  $r/D \gg 1$ , the induced velocity reduces to that of a dipole. Substituting the slug-model value of circulation  $\Gamma = u_j^2 t$ , the induced velocity is approximately given by

$$\mathbf{u}_{ind}(r,z=0) \approx -u_j \frac{3}{32} \frac{u_j t}{D} \left(\frac{D}{r}\right)^3 \mathbf{\hat{z}}$$

This form indicates that the induced velocity is set by the jet velocity, scaled by the nozzle stroke ratio  $u_j t/D$  and a normalized distance r/D. While the asymptotic behavior of  $u_j$  is not formally valid for the near-field regime that I am studying (1.5 < r/2a < 3), this form well approximates the true behavior, and reveals a simple scaling that provides insight into the problem at hand.

Given the induced velocity field, the associated pressure that would be required to establish such a field can be determined by the stagnation pressure:

$$p_{ind} \sim \rho u_{ind}^2$$
  
 $\sim \rho u_j^2 \left(\frac{u_j t}{D}\right)^2 \left(\frac{D}{\Delta}\right)^6$ 

Using this vortex-induction argument, I suggest that for a two-jet system, the additional pressure scale that is introduced by a second nozzle is given by the induced pressure

$$p_{ind} \sim C \widetilde{\Delta}^{-6}$$

### Appendix C

# Additional data from two-nozzle experiments



Figure C-1: Time-series evolution of CV position and volume. This plot shows data calculated from the wake of nozzle 1 at a few values of  $\widetilde{\Delta}$ .



Figure C-2: Time-series evolution of vortex bubble geometry (a, b) and growth rates  $(\dot{a}, \dot{b})$ . Time derivatives in these plots are calculated numerically. This plot shows data calculated from the wake of nozzle 1 at a few values of  $\tilde{\Delta}$ .



**Figure C-3:** Non-normalized CV velocity, as a function of  $\tilde{\Delta}$ . Blue data points are calculated from wake measurements at nozzle 1. Red data points are calculated from nozzle 2.



Figure C-4: Non-normalized vortex bubble geometry evolution (minor axis growth rate  $\dot{a}$  and major axis growth rate  $\dot{b}$ ), as a function of  $\tilde{\Delta}$ . Blue data points are calculated from wake measurements at nozzle 1. Red data points are calculated from nozzle 2.



Figure C-5: Non-normalized average momentum and pressure contributions to thrust, as a function of  $\tilde{\Delta}$ . Blue data points are calculated from wake measurements at nozzle 1. Red data points are calculated from nozzle 2.



Figure C-6: Time-series contributions to total thrust - pressure contribution (left) and inertial contribution (right). This plot shows data calculated from the wake of nozzle 1 at a few values of  $\tilde{\Delta}$ .



Figure C-7: Thrust calculated from the wake of nozzle 1 at different values of  $\tilde{\Delta}$ .

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